Full-Duplex Cooperative Rate Splitting Multiple Access with Imperfect CSI and Imperfect SIC

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***Abstract*—Rate splitting multiple access (RSMA) is one of the promising multiple access techniques in 6G. One of the limitations of RSMA is that the rate is limited by the worst user's rate. Cooperative rate splitting multiple access (C-RSMA) has become an essential technique to overcome this limitation. In this work, a downlink full-duplex C-RSMA with imperfect channel state information (ICSI) and imperfect successive interference cancellation (ISIC) is considered. Full-duplex (FD) relaying is used to fully utilize the channel in order to enhance the system performance. The objective is to maximize the minimum rate of the users, while ensuring that the transmission latency for each user's rate does not exceed a specified threshold. Success convex approximation (SCA) is used to optimize the precoding matrix, common rate allocation, and the relay power. Numerical results show that the proposed system improves the Max-Min rate compared to the other systems.**

***Index Terms*—Sixth generation, wireless communications, rate splitting multiple access, cooperative communication, imperfect channel state information, imperfect successive interference cancellation**

I. Introduction

Rate-Splitting Multiple Access (RSMA) has attracted significant attention from both industry and academia due to its ability to meet the requirements of 6G and beyond, including high spectral efficiency and ultra-reliable low-latency communication [1]. It is able to effectively managing interference by splitting the message of each user into a common part and a private part. The common parts of the users are combined together and encoded into one stream while the private parts are encoded independently. At the receiver side, each user starts by decoding the common stream first, then the decoded common stream can be removed from the received signal by using successive interference cancellation (SIC). After that, each user decodes its own private stream. This message split allows to control the interference between the streams and enhance the system performance [2]. A limitation of RSMA is that the rate of the common stream is constrained by the user with the lowest achievable rate for that stream. Therefore, RSMA has been integrated with cooperative user relaying to realize cooperative RSMA (C-RSMA). In C-RSMA the strongest user acts as relay to forward the common stream to the rest of users [3], [4].

The authors in [5]–[7] introduce C-RSMA and show how this system outperforms both non-cooperative RSMA as well as other multiple access techniques. However, these papers consider half-duplex (HD) cooperative communications systems which suffer from inefficient usage of time resources and latency. Full-duplex (FD) is introduced to fully utilize the time resources and improve the performance of the system. In [8], both full-duplex cooperative-non-orthogonal-multiple-access (C-NOMA) and full-duplex C-RSMA schemes for uplink user cooperation are proposed. It is concluded that their performance is better than those without cooperation, and C-RSMA has a better performance than C-NOMA. In [9], the authors consider a two-user uplink C-RSMA using full duplex, and aim to maximize the minimum signal-to-interference-plus-noise ratio (SINR) by jointly optimizing the beamforming at the base station (BS) and device transmit power. The optimization problem is solved using successive convex approximation (SCA) and a difference of convex (DC)-based approach to approximate the non-convex problem to a convex one. It was shown by simulation results that the proposed scheme can outperform cooperative non-orthogonal multiple access (C-NOMA) and other baseline schemes. Also, full-duplex C-RSMA is addressed in [10] to aid heterogeneous networks (HetNets), as the achievable rate is limited by cross-layer interference and the rate of the user with the worst channel condition. Furthermore, full-duplex C-RSMA is used to improve the weighted sum of the average secrecy rate of remote multi-cast groups as introduced in [11], and to maximize the sum rate of network in [12].

All the above-mentioned papers consider perfect channel state information (CSI), which is usually impractical. So, imperfect channel state information (ICSI) should be used in the communication systems to simulate the real life systems. RSMA was proved to be more robust than the other multiple access technique in case of ICSI [13]–[15]. Another practical challenge that is not considered in the above-mentioned papers is the imperfect SIC (ISIC). In [16], the authors aim to maximize the weighted sum rate (WSR) and weighted energy efficiency (WEE) under imperfect SIC. In addition, other papers combine these two practical challenges to simulate the real-life scenarios, but without considering C-RSMA [17].

Furthermore, one of the characteristics in 6G communications is ultra-low latency. RSMA is used to enhance latency when compared to other conventional multiple access techniques. In [18], RSMA is used to minimize the average latency of a caching network. The simulation results show that RSMA outperforms all the other existing multiple access techniques in minimizing the latency.

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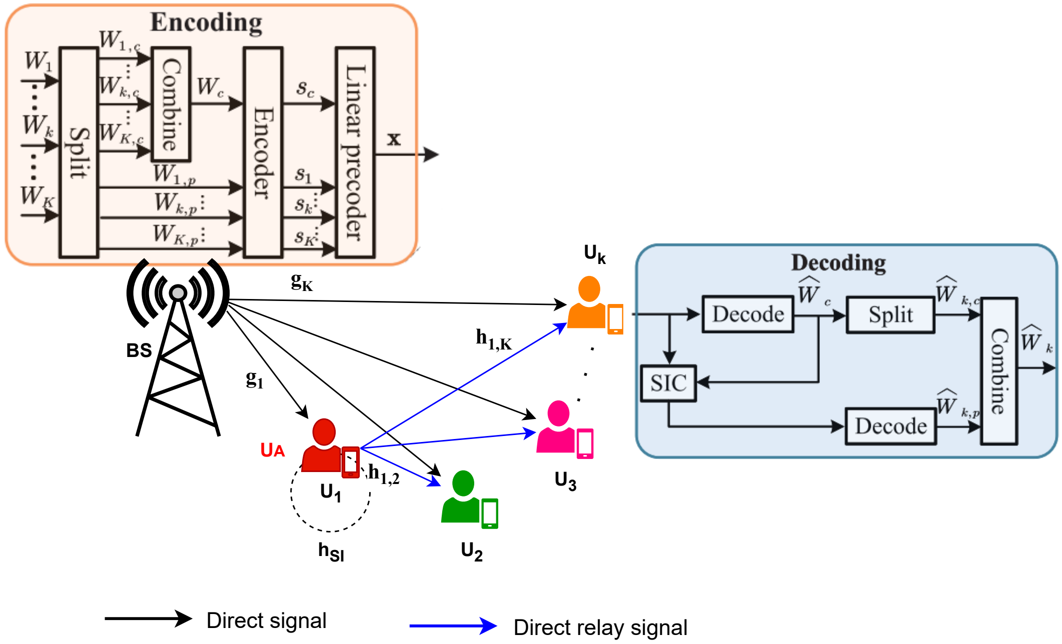

Fig. 1: Full-duplex C-RSMA

It can be seen from the aforementioned investigations that full-duplex multi-user cooperative rate splitting multiple access (FD C-RSMA) system under both imperfect CSI and imperfect SIC is not yet deeply analyzed. The objective in this paper is to maximize the minimum rate of the users in a downlink full-duplex C-RSMA system with imperfect CSI and imperfect SIC while taking into consideration that the latency does not exceed certain threshold.

Our main contributions can be summarized as follows:

- We integrate the RSMA with full-duplex user relaying to improve the maximum minimum rate (Max-Min rate) of multi-user communications system under imperfect CSI and imperfect SIC.
- We formulate an optimization problem that maximizes the minimum rate of the users, by jointly optimizing the transmit precoding matrix, the common rate allocation, and the relay power while restricting the latency to be less than a threshold value.
- We solve the optimization problem using success convex approximation. The performance of the system is compared with half-duplex C-RSMA and other systems to show the superiority of this system.

II. System and Channel Model

We consider a downlink full-duplex cooperative RSMA (FD C-RSMA) wireless communication system, as shown in Fig. 1. Multi-user multiple input single output (MISO) is considered, where a base station (BS) is equipped with N_t transmit antennas that serves K users with single antenna. The channel between the BS and user-k is denoted by $\mathbf{g}_k \in \mathbb{C}^{N_t \times 1}$, and the channel between two different users is denoted by $h_{i,j}$ where $i \neq j$, and $i, j \in K$. We also consider imperfect CSI and imperfect SIC for more realistic impairments in the system. The imperfect CSI can be considered as:

$$\mathbf{g}_k = \hat{\mathbf{g}}_k + \mathbf{e}_k \tag{1}$$

where $\hat{\mathbf{g}}_k$ is the estimate of the channel $\mathbf{g}_k$, and $\mathbf{e}_k$ is the channel estimation error between BS and user-k which is a random variable with zero mean and variance $\sigma^2_{e,k}$. Similarly, the imperfect channel between the users is expressed as:

$$h_{ij} = \hat{h}_{i,j} + e_{i,j} \tag{2}$$

It is assumed that the nearest user to the BS, has the best channel condition among the rest of the users as it has the highest channel gain $\parallel \hat{\mathbf{g}}_k \parallel^2$, and acts as a full-duplex (FD) relay to forward the signal to the rest of users, this user is denoted by (U_A). As for the imperfect SIC, it causes residual interference when decoding the common stream which is represented by β, where $0<\beta<1$.

III. Problem Formulation

The message of user k, W_k, is split into two parts: common part $W_{c,k}$ and private part $W_{p,k}$. All the common parts are combined together and encoded into a common stream s_c using a common codebook, while the private parts are encoded independently and transformed to private stream s_k. The common stream is intended for all users but the private streams are intended for their corresponding users. Then, the signal transmitted by BS can be written as:

$$\mathbf{x} = \mathbf{p}_c \mathbf{s}_c + \sum_{k=1}^{K} \mathbf{p}_k \mathbf{s}_k \tag{3}$$

where $\mathbf{p}_k \in \mathbb{C}^{N_t \times 1}$ is the precoding vector of user-k and it

$$SINR_A^C = \frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_c \|^2}{\sum_{i=1}^{K}(\| \hat{\mathbf{g}}_A^H \mathbf{p}_i \|^2 + \sigma_{e,A}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,A}^2 \| \mathbf{p}_c \|^2 + P_r \| h_{SI} \|^2 + N_0}), \quad (4)$$

$$SINR_A^P = \frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_A \|^2}{\sum_{\substack{i=1 \\ i \neq A}}^{K}(\| \hat{\mathbf{g}}_A^H \mathbf{p}_i \|^2 + \sigma_{e,A}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,A}^2 \| \mathbf{p}_A \|^2 + \beta \| \hat{\mathbf{g}}_A^H \mathbf{p}_c \|^2 + \beta\sigma_{e,A}^2 \| \mathbf{p}_c \|^2 + P_r \| h_{SI} \|^2 + N_0}, \quad (5)$$

$$SINR_k^C = \frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_c \|^2}{\sum_{i=1}^{K}(\| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + \sigma_{e,k}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,k}^2 \| \mathbf{p}_c \|^2 + N_0}), \quad (6)$$

$$SINR_k^P = \frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_k \|^2}{\sum_{\substack{i=1 \\ i \neq k \\ i \neq A}}^{K}(\| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + \sigma_{e,k}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,k}^2 \| \mathbf{p}_k \|^2 + \beta \| \hat{\mathbf{g}}_k^H \mathbf{p}_c \|^2 + \beta\sigma_{e,k}^2 \| \mathbf{p}_c \|^2 + N_0}, \quad (7)$$

is one column of the matrix $\mathbf{P} \in \mathbb{C}^{N_t \times (K+1)}$. Assume that $\mathbb{E}[\mathbf{ss}^H] = \mathbf{I}$ and the transmit power constraint is $tr(\mathbf{PP}^H) \leq P_t$, where P_t is the maximum transmit power of the BS. Then, the received signal at the relay user (U_A) is given by:

$$y_A = \mathbf{g}_A^H \mathbf{x} + h_{SI}\sqrt{P_r}\hat{s}_c + n = (\hat{\mathbf{g}}_A^H + \mathbf{e}_A^H)\mathbf{x} + h_{SI}\sqrt{P_r}\hat{s}_c + n \quad (8)$$

where P_r is user relay transmit power, h_{SI} is the self interference channel at the user relay due to full-duplex communication which follows $h_{SI} \sim \mathcal{CN}(0, \sigma_{SI}^2)$, and n is complex additive white Gaussian noise (AWGN) at the users with power spectral density denoted by N_0. While $\hat{s}_c$ represents the delayed version of the common stream forwarded by the user relay, $\hat{s}_c = s_c(i-\tau)$, where i is the i^{th} time slot and τ is the processing time to decode the common stream at the user relay. It is important to note that τ is insignificant compared to the duration of one time slot, thus $\hat{s}_c = s_c$ [19]. When user k receives the transmitted signal, it first decodes the common stream s_c by treating the private streams as interference. Then, the achievable rate of decoding the common stream at the relay user (U_A) is

$$c_A = \log_2(1 + SINR_A^C), \quad (9)$$

where $SINR_A^C$ is calculated according to Eq. (4). In the denominator of $SINR_A^C$, the first term ($\| \hat{\mathbf{g}}_A^H \mathbf{p}_i \|^2$) represents the interference from the estimated channel of the private messages of the other users, the second term ($\sigma_{e,A}^2 \| \mathbf{p}_i \|^2$) represents the interference from the channel estimation error of the private messages of the users, the third term ($\sigma_{e,A}^2 \| \mathbf{p}_c \|^2$) represents the interference of the channel estimation error of the common stream, and the fourth term ($P_r \| h_{SI} \|^2$) represents the residual interference at the user relay.

After decoding the common stream, successive interference cancellation (SIC) is applied to remove it. However, due to imperfect SIC, the common stream is not perfectly removed, leaving residual interference. This residual interference from the common stream affects the decoding of the private stream for each user. Hence, the rate of decoding the private stream at the relay user (U_A) is

$$r_A = \log_2(1 + SINR_A^P), \quad (10)$$

where $SINR_A^P$ is calculated following Eq. (5). In the denominator of $SINR_A^P$, the fourth term ($\beta \| \hat{\mathbf{g}}_A^H \mathbf{p}_c \|^2$) and the fifth term ($\beta\sigma_{e,A}^2 \| \mathbf{p}_c \|^2$) represent the residual interference due to the imperfect SIC of the common stream.

At the same time, the non-relaying users receive two signals in the same time slot, one from the BS; due to the direct transmission; and the other one is the common stream from the relaying user; due to cooperative transmission. Then, the received signal at the non-relaying users is

$$y_k = \underbrace{(\hat{\mathbf{g}}_k^H + \mathbf{e}_k^H)\mathbf{x}}_{\substack{\text{recieved} \\ \text{signal from BS}}} + \underbrace{(\hat{h}_{A,k} + e_{A,k})\sqrt{P_r}s_c}_{\substack{\text{recieved common} \\ \text{stream from user relay}}} + n, \quad (11)$$

for $k \in K$ and $k \neq A$, where $\mathrm{h}_{A,k}$ is the channel between the relay user (U_A) and user-k, and $e_{A,k}$ is the channel estimation error between the relay user (U_A) and user-k which is a random variable with zero mean and variance $\sigma_{e,Ak}^2$. We assume that the streams from the BS and from user relay U_A can be fully resolved at non-relaying users so that they can be appropriately co-phased and merged by maximal ratio combining (MRC) [19]. Thus, the rate of decoding the common stream for non-relaying users is given by

$$c_k = \log_2(1 + \frac{P_r \| \hat{h}_{A,k} \|^2}{\sigma_{e,Ak}^2 P_r + N_0} + SINR_k^C), \quad (12)$$

for $k \in K$, and $k \neq A$, where $SINR_k^C$ is calculated using Eq. (6). After that, the common stream is removed from the received signal after decoding it using imperfect SIC. Hence, the rate of decoding the private stream of non-relaying users is

$$r_k = \log_2(1 + SINR_k^P), \quad (13)$$

where $SINR_k^P$ is calculated using Eq. (7).

In order to guarantee that all users can decode the common stream s_c successfully, the achievable rate to decode the common stream should be

$$R_c = \min(c_1, c_2, ..., c_K). \quad (14)$$

Note that, R_c is shared by all users, as the common stream contains information from each user, thus it should satisfy

$$\sum_{i=1}^{K} a_i \leq R_c \quad (15)$$

where a_i is the portion of the common stream allocated to each user. Hence, the total achievable rate of user k is

$$R_{k,tot} = r_k + a_k, \forall k \in K. \tag{16}$$

Low latency is one of the critical characteristics that should be addressed by 6G in order to support real-time technologies like the tactile internet including Metaverse and VR applications [20]–[22]. To ensure high system performance, latency should be less than certain threshold value t_{min}. The transmission latency of user k (T_k) [17] is given by

$$T_k = \frac{L_k}{R_{k,tot}} \tag{17}$$

where $R_{k,tot}$ is the total achievable rate of user-k and L_k is the normalized data size which is given by $L_k = \frac{N_k}{B}$, where N_k is data size and B is the bandwidth [23].

The objective is to maximize the minimum rate (Max-Min rate) of all users in order to ensure the fairness among users. This can be done by jointly optimizing the precoding matrix $\mathbf{P}$, the rate portion of common stream for each user $\boldsymbol{a}$, and the relay power P_r.

The optimization problem can be formulated as

$$\max_{\mathbf{P},\boldsymbol{a},P_r} \quad \min \ R_{k,tot} \tag{18}$$

$$s.t: \sum_{i=1}^{K} a_i \leq R_c, \tag{18a}$$

$$a_k \geq 0, \forall k \in K, \tag{18b}$$

$$tr(\mathbf{P}\mathbf{P}^H) \leq P_t, \tag{18c}$$

$$\frac{L_k}{R_{k,tot}} \leq t_{min}, \tag{18d}$$

$$P_r \leq P_{r,max}, \tag{18e}$$

where constraint (18a) guarantees that each user is able to decode the common stream. Constraint (18b) shows that the portion of the common rate allocated to each user should be positive value, while (18c) and (18e) refer to the power budget at the BS and the user relay respectively, and (18d) guarantees the user's requirement of transmission latency. The objective function, constraint (18a), and constraint (18d) are non-convex. Thus, success convex approximation (SCA) is used to solve this optimization problem.

IV. Optimization Technique

In this section, we focus on optimizing the precoding matrix, the common rate, and the relay power. Let v denotes the minimum rate of all users. Then, the optimization problem (18), can be reformulated as follows

$$\max_{\mathbf{P},\boldsymbol{a},P_r,v} \quad v \tag{19}$$

$$s.t: r_k + a_k \geq v, \forall k \in K, \tag{19a}$$

$$r_k + a_k \geq \frac{L_k}{t_{min}} \tag{19b}$$

$$(18a), (18b), (18c), (18e)$$

It is noticed that problem (19) is still non-convex due to constraints (18a), (19a) and (19b), so we introduce slack variables to solve this non-convexity. Let $\boldsymbol{\gamma} = [\gamma_1, ..., \gamma_K]$, $\boldsymbol{\gamma_c} = [\gamma_{c,1}, ..., \gamma_{c,K}], \forall k \in K$, and $\boldsymbol{\gamma_c^{(2)}} = [\gamma_{c,1}^{(2)}, ..., \gamma_{c,K}^{(2)}], k \in K, k \neq A$, denotes the signal-to-interference-plus noise (SINR) vector for the private stream, the common stream due to direct transmission, and the common stream due to cooperative transmission, respectively. Also $\boldsymbol{\kappa} = [\kappa_1, ..., \kappa_K]$, $\boldsymbol{\kappa_c} = [\kappa_{c,1}, ..., \kappa_{c,K}], \forall k \in K$, denotes the interference-plus noise (INR) vector for the private stream, and the common stream, respectively.

Now, the optimization problem (19) can be rewritten as

$$\max_{\mathbf{P},\boldsymbol{a},p_r,v,\boldsymbol{\gamma},\boldsymbol{\gamma_c},\boldsymbol{\gamma_c^{(2)}}} \quad v \tag{20}$$

$$s.t: \log_2(1+\gamma_k) + a_k \geq v, \forall k \in K, \tag{20a}$$

$$\log_2(1+\gamma_k) + a_k \geq \frac{L_k}{t_{min}}, \forall k \in K, \tag{20b}$$

$$\log_2(1+\gamma_{c,A}) \geq \sum_{i=1}^{K} a_i, \tag{20c}$$

$$\log_2(1+\gamma_{c,k}^{(2)} + \gamma_{c,k}) \geq \sum_{i=1}^{K} a_i, k \in K, k \neq A \tag{20d}$$

$$\frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_A \|^2}{\kappa_A} \geq \gamma_A, \tag{20e}$$

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_k \|^2}{\kappa_k} \geq \gamma_k, k \in K, k \neq A, \tag{20f}$$

$$\frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_c \|^2}{\kappa_{c,A}} \geq \gamma_{c,A}, \tag{20g}$$

$$\frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_c \|^2}{\kappa_{c,k}} \geq \gamma_{c,k}, k \in K, k \neq A, \tag{20h}$$

$$\frac{P_r \| \hat{h}_{A,k} \|^2}{\sigma_{e,Ak}^2 P_r + N_0} \geq \gamma_{c,k}^{(2)}, k \in K, k \neq A, \tag{20i}$$

$$(18b), (18c), (18e)$$

where the expression of κ_A, κ_k, $\kappa_{c,A}$ and $\kappa_{c,k}$ are given by

$$\kappa_A \geq \sum_{\substack{i=1 \\ i\neq A}}^{K} (\| \hat{\mathbf{g}}_A^H \mathbf{p}_i \|^2 + \sigma_{e,A}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,A}^2 \| \mathbf{p}_A \|^2 + \beta \| \hat{\mathbf{g}}_A^H \mathbf{p}_c \|^2 + \beta \sigma_{e,A}^2 \| \mathbf{p}_c \|^2 + P_r \| h_{SI} \|^2 + N_0, \tag{21}$$

$$\kappa_k \geq \sum_{\substack{i=1 \\ i\neq k \\ i\neq A}}^{K} (\| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + \sigma_{e,k}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,k}^2 \| \mathbf{p}_k \|^2 + \beta \| \hat{\mathbf{g}}_k^H \mathbf{p}_c \|^2 + \beta \sigma_{e,k}^2 \| \mathbf{p}_c \|^2 + N_0, \tag{22}$$

$$\kappa_{c,A} \geq \sum_{i=1}^{K} (\| \hat{\mathbf{g}}_A^H \mathbf{p}_i \|^2 + \sigma_{e,A}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,A}^2 \| \mathbf{p}_c \|^2 + P_r \| h_{SI} \|^2 + N_0, \tag{23}$$

$$\kappa_{c,k} \geq \sum_{i=1}^{K}(\| \hat{\mathbf{g}}_k^H \mathbf{p}_i \|^2 + \sigma_{e,k}^2 \| \mathbf{p}_i \|^2) + \sigma_{e,k}^2 \| \mathbf{p}_c \|^2 + N_0, \tag{24}$$

Here, (20e) - (20h) are still non-convex; but they follow a generic form as $f(x, y) = \frac{\|y\|^2}{x}, \forall y \in \mathbb{C}, \forall x \in \mathbb{R}^+$, that can be approximated using a lower bounded concave approximation as mentioned in [5]. The function $f(x, y)$ can be approximated on point $x^{(l)}$,$y^{(l)}$ in order to solve non-convexity in iterative manner. Then, the approximated generic equation is given by

$$f(x, y) \geq F(x, y; x^{(l)}, y^{(l)}) = \frac{2\mathcal{R}\{y^{(l)*} y\}}{x^{(l)}} - \frac{\| y^{(l)} \|^2}{(x^{(l)})^2} x, \tag{25}$$

Using Eq. (25), Eq. (20e)-(20h) can be approximated by (26) and (29):

$$\frac{2\mathcal{R}\{(\mathbf{p}_A^{(l)})^H \hat{\mathbf{g}}_A \hat{\mathbf{g}}_A^H \mathbf{p}_A\}}{\kappa_A^{(l)}} - \frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_A^{(l)} \|^2 \kappa_A}{(\kappa_A^{(l)})^2} \geq \gamma_A, \tag{26}$$

$$\frac{2\mathcal{R}\{(\mathbf{p}_k^{(l)})^H \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \mathbf{p}_k\}}{\kappa_k^{(l)}} - \frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_k^{(l)} \|^2 \kappa_k}{(\kappa_k^{(l)})^2} \geq \gamma_k, k \in K, k \neq A, \tag{27}$$

$$\frac{2\mathcal{R}\{(\mathbf{p}_c^{(l)})^H \hat{\mathbf{g}}_A \hat{\mathbf{g}}_A^H \mathbf{p}_c\}}{\kappa_{c,A}^{(l)}} - \frac{\| \hat{\mathbf{g}}_A^H \mathbf{p}_c^{(l)} \|^2 \kappa_{c,A}}{(\kappa_{c,A}^{(l)})^2} \geq \gamma_{c,A}, \tag{28}$$

$$\frac{2\mathcal{R}\{(\mathbf{p}_c^{(l)})^H \hat{\mathbf{g}}_k \hat{\mathbf{g}}_k^H \mathbf{p}_c\}}{\kappa_{c,k}^{(l)}} - \frac{\| \hat{\mathbf{g}}_k^H \mathbf{p}_c^{(l)} \|^2 \kappa_{c,k}}{(\kappa_{c,k}^{(l)})^2} \geq \gamma_{c,k}, k \in K, k \neq A, \tag{29}$$

where $\mathcal{R}$ represents the real number. Furthermore, Eq. (20i) is non-convex. Thus, using cross multiplication it can be rewritten as

$$P_r \| \hat{h}_{A,k} \|^2 - \gamma_{c,k}^{(2)} N_0 \geq \sigma_{e,Ak}^2 (P_r)(\gamma_{c,k}^{(2)}) \tag{30}$$

However, Eq. (30) is still non-convex and can be converted to a convex one by converting the term after the inequality in (30) using difference of squares identity as follows

$$P_r \| \hat{h}_{A,k} \|^2 - \gamma_{c,k}^{(2)} N_0 \geq \frac{\sigma_{e,Ak}^2}{4}((P_r + \gamma_{c,k}^{(2)})^2 - (P_r - \gamma_{c,k}^{(2)})^2) \tag{31}$$

Finally, the optimization problem (19) at iteration (l) can be reformulated as

$$\max_{\mathbf{P}, \boldsymbol{a}, P_r, v, \boldsymbol{\gamma}, \boldsymbol{\gamma_c}, \boldsymbol{\gamma_c^{(2)}}, \boldsymbol{\kappa}, \boldsymbol{\kappa_c}} \quad v \tag{32}$$
$$s.t: (18b), (18c), (18e), (20a), (20b), (20c), (20d),$$
$$(21), (22), (23), (24), (26), (27), (28), (29), (31).$$

Problem (32) is now a convex problem that can be solved iteratively using success convex approximation (SCA), and the details of this solution is illustrated in the iterative **Algorithm 1**. The algorithm starts with assuming initial values for the variables $\mathbf{P}, \boldsymbol{\kappa}$, and $\boldsymbol{\kappa_c}$. These variables are used to solve (32). We get the updated values of the mentioned parameters and the objective function. The algorithm continues iteratively updating these variables till convergence. Then, we compare the new optimized Max-Min rate (objective function) with the previous value if it is greater than certain tolerance value (ϵ), this process will be repeated until convergence.

Algorithm 1: Joint precoding matrix, common rate allocation, and relay power optimization algorithm

Given : tolerance ϵ
Initialize: $\mathbf{P}^{(0)}, \boldsymbol{\kappa}^{(0)}, \boldsymbol{\kappa_c}^{(0)}, v^{(0)} = 0$;
$l = 0$;
Solve (32) using $\mathbf{P}^{(0)}$,$\boldsymbol{\kappa}^{(0)}$,$\boldsymbol{\kappa_c}^{(0)}$;
Find the optimal value for the objective function v^* and optimal variables $\mathbf{P}^{(*)}$,$\boldsymbol{\kappa}^{(*)}$,$\boldsymbol{\kappa_c}^{(*)}$;
$l = l + 1$;
Update the old variables $v^{(1)} \leftarrow v^{(*)}$, $\mathbf{P}^{(1)} \leftarrow \mathbf{P}^{(*)}$, $\boldsymbol{\kappa}^{(1)} \leftarrow \boldsymbol{\kappa}^{(*)}$, $\boldsymbol{\kappa_c}^{(1)} \leftarrow \boldsymbol{\kappa_c}^{(*)}$;
while $| v^{(l)} - v^{(l-1)} | > \epsilon$ **do**
  $l = l + 1$;
  Solve (32) using $\mathbf{P}^{(l-1)}$, $\boldsymbol{\kappa}^{(l-1)}$,$\boldsymbol{\kappa_c}^{(l-1)}$;
  Find the optimal value for the objective function v^* and optimal variables $\mathbf{P}^{(*)}$, $\boldsymbol{\kappa}^{(*)}$, $\boldsymbol{\kappa_c}^{(*)}$;
  Update the old variables: $v^{(l)} \leftarrow v^{(*)}$, $\mathbf{P}^{(l)} \leftarrow \mathbf{P}^{(*)}$, $\boldsymbol{\kappa}^{(l)} \leftarrow \boldsymbol{\kappa}^{(*)}$, $\boldsymbol{\kappa_c}^{(l)} \leftarrow \boldsymbol{\kappa_c}^{(*)}$;
end

V. NUMERICAL RESULTS

Numerical results are obtained to evaluate the performance of the full-duplex C-RSMA system in terms of the maximum minimum rate (Max-Min) rate. The performance of the proposed system is compared with the following systems:

- **FD C-RSMA ICSI ISIC:** Full-duplex cooperative RSMA with both imperfect CSI and SIC.
- **FD C-RSMA ICSI PSIC:** Full-duplex cooperative RSMA with imperfect CSI and perfect SIC.
- **FD C-RSMA PCSI ISIC:** Full-duplex cooperative RSMA with perfect CSI and imperfect SIC.
- **FD C-RSMA PCSI PSIC:** Full-duplex cooperative RSMA with both perfect CSI and SIC.
- **HD C-RSMA ICSI ISIC:** Half-duplex cooperative RSMA with both imperfect CSI and SIC.
- **HD C-RSMA ICSI PSIC:** Half-duplex cooperative RSMA with imperfect CSI and perfect SIC.
- **HD C-RSMA PCSI ISIC:** Half-duplex cooperative RSMA with perfect CSI and imperfect SIC.
- **HD C-RSMA PCSI PSIC:** Half-duplex cooperative RSMA with both perfect CSI and SIC.
- **RSMA ICSI ISIC:** RSMA with both imperfect CSI and SIC without user cooperation.
- **RSMA ICSI PSIC:** RSMA with imperfect CSI and perfect SIC, without user cooperation.

TABLE I: Simulation parameters

Parameter	Value
BS antenna, N_t	2
Power transmitted, P_t	12 dB
Relay power, $P_{r,max}$	5 dB
AWGN power, N_0	-40 dB
Channel error variance, $\sigma_{e,k}^2$	-40 dB
SIC imperfection factor, β	0.2
SI power, σ_{SI}^2	-30 dB
Latency threshold, t_{min}	1ms
Path-loss exponent	2
Tolerance, ϵ	0.001
Number of users, K	4
Number of reflecting elements, N	4
Base station (BS) position	(0,0)
First user, U1	(40,0)
Second user, U2	(50,0)
Third user, U3	(60,0)
Fourth user, U4	(80,0)

- **RSMA PCSI ISIC:** RSMA with perfect CSI and imperfect SIC, without user cooperation.
- **RSMA PCSI PSIC:** RSMA with both perfect CSI and SIC, without user cooperation.

In our simulation, user 1 will be the user relay (U_A=U_1), as it is the one with best channel condition. We assume that the channel estimation error of all users ($\sigma_{e,k}^2$) is the same. The simulation parameters are given in **Table** I.

Fig. 2 shows the convergence of the SCA algorithm. It plots the Max-Min rate versus the number of iterations. It can be seen that all the mentioned systems converge within the first 10 iterations.

Fig. 3 shows the Max-Min rate of the different systems under study versus the transmitted power. It is shown that the Max-Min rate of the users increases as the power increases. It is noticeable that the proposed system is superior over all the other systems, as this system uses full-duplex that transmit and receives the signal in same time slot. The Max-Min rate increases by 8 % using FD C-RSMA compared with HD C-RSMA at transmitted power 16 dB. It is also shown that the systems using cooperative communications have higher rate than the non-cooperative systems, which shows that cooperative technology can overcome the limitation of RSMA. It is also shown that the systems with perfect CSI and perfect SIC has better performance than their corresponding ones with imperfect CSI and imperfect SIC.

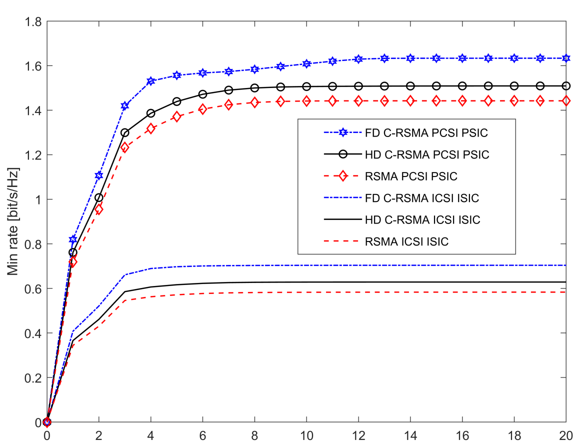

Fig. 2: Max-Min rate vs. number of iterations

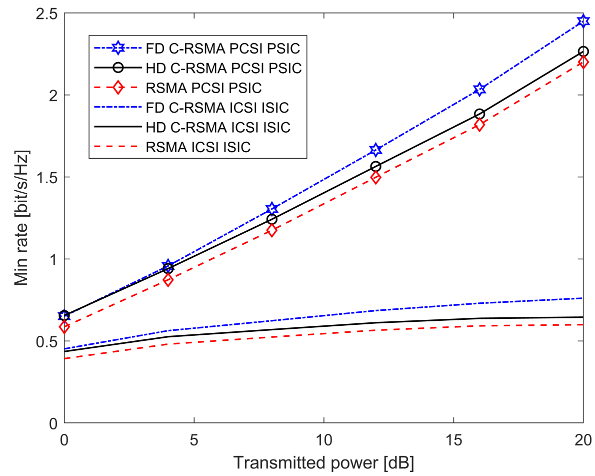

Fig. 3: Max-Min rate vs. transmitted power

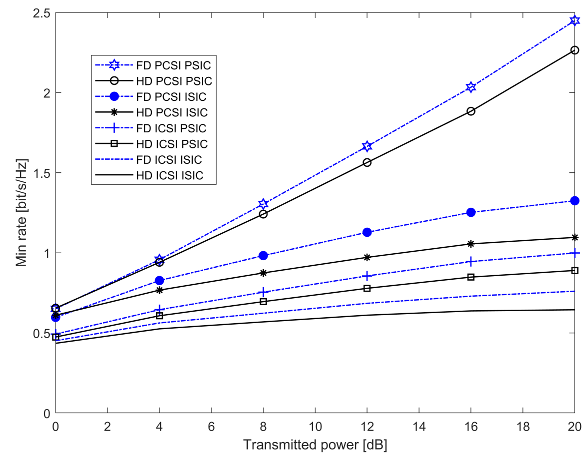

Fig. 4: Max-Min rate vs. transmitted power with different practical impairments

Fig. 4 shows the Max-Min rate versus transmitted power for FD C-RMSA and HD C-RSMA with different practical impairments. It is shown that the systems using full-duplex communication have higher rate than their corresponding ones using half-duplex due to exploiting the time resource. It is worth noting that the channel imperfection has significant effect on the system performance, as the figure shows that the system with imperfect CSI and perfect SIC has lower rate than the system with perfect CSI and imperfect SIC and this implies that imperfect CSI has the dominant effect on the performance of the system degradation.

Fig. 5 shows the Max-Min rate versus the power transmitted

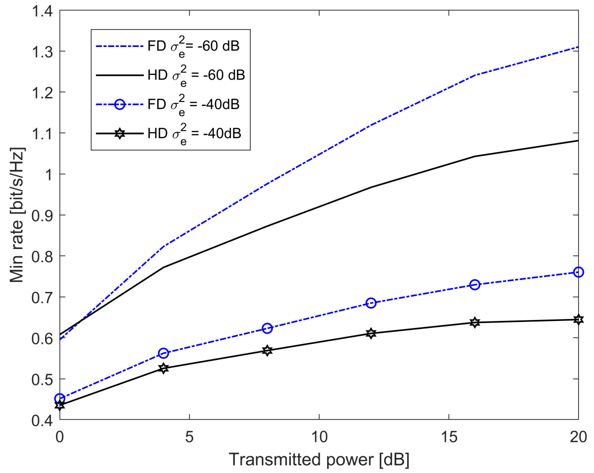

Fig. 5: Max-Min rate vs. transmitted power with different channel estimation error variance (σ_e^2)

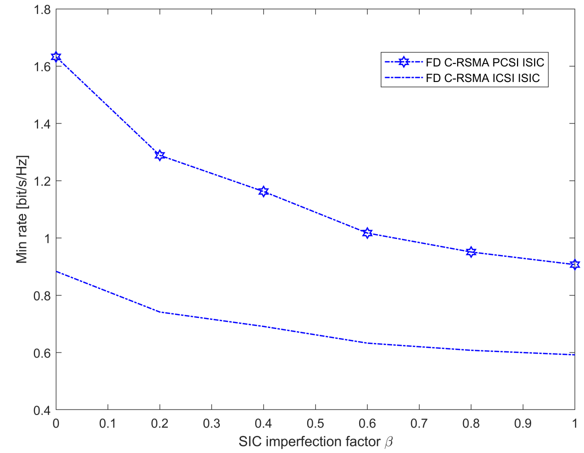

Fig. 7: Max-Min rate vs. SIC imperfection factor β for FD C-RSMA with $P_{r,max} = P_t$

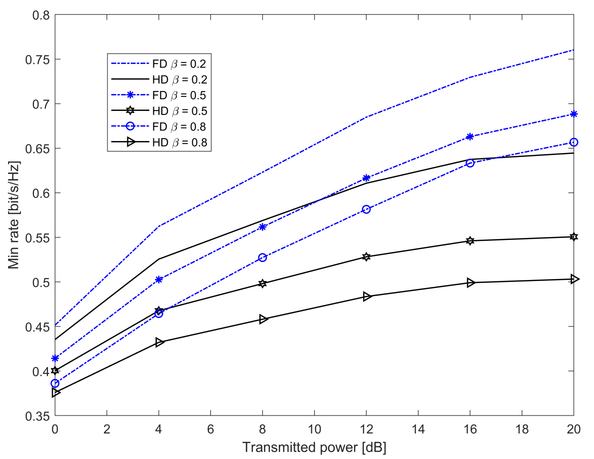

Fig. 6: Max-Min rate vs. power transmitted with different β

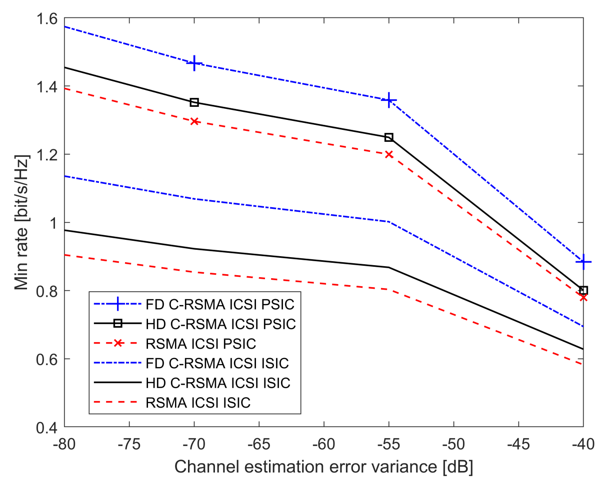

Fig. 8: Max-Min rate vs. channel estimation error factor

for FD C-RSMA and HD C-RSMA for different channel estimation error variance (σ_e^2). It is shown that the system with higher estimation error variance has lower Max-Min rate when compared with other systems. This is because as estimation error increases, the interference and error increase and thus the rate decreases. It is also shown that the systems using FD relaying have higher rate than that with HD relaying.

Fig. 6 shows the Max-Min rate for different SIC imperfection β versus the transmitted power. This figure compares between FD C-RSMA and HD C-RSMA. It is shown that the systems with higher SIC imperfection factor have lower rate. This is because as β increases, the interference increases and the rate decreases.

Fig. 7 shows the Max-Min rate versus SIC imperfection β for FD C-RSMA. It is shown that as β increases the rate decreases; because as β increases, the error in decoding the common stream increases which increases the interference and decreases the Max-Min rate. It is also shown that FD C-RSMA with perfect CSI has higher rate than that with imperfect CSI.

Fig. 8 shows the Max-Min rate versus channel estimation error factor for FD C-RSMA, HD C-RSMA and RSMA with both perfect CSI and SIC, and imperfect ones. It is shown that as channel estimation error increases, the rate decreases due to increasing interference. It is also shown that FD C-RSMA has the superiority over the other systems both in case of ICSI and ISIC or perfect ones.

VI. Conclusion

In this paper, a downlink full-duplex multi-user C-RSMA has been studied under imperfect CSI and imperfect SIC and also while taking in consideration the latency constraint. The objective is to maximize the minimum rate of all users by optimizing the precoding matrix, common rate allocation and the relay power. Success convex approximation is used to solve the optimization problem iteratively. In order to show the superiority of the proposed system, it is compared with HD C-RMSA and other systems. Numerical results ensure that the proposed system outperforms all the other mentioned systems. As for the future work, the system can be extended to use 2-layer RSMA principle, and compare its performance with 1-layer RSMA.

Acknowledgments

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