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Centralized vs. Distributed Frequency Assignment in Frequency Hopping (Cognitive Radio) Cellular Networks

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Abstract

Frequency Assignment is an important approach to mitigate multicell interference in cellular systems. In this paper, we consider the frequency assignement in such systems with frequency hopping. In particular we focus our attention on cognitive radio cellular systems as one of the very promising future access technologies, taking IEEE 802.22 as an example, which is currently under standardization. While the optimal frequency assignment for such a system is conceptually straightforward – as well as computationally complex – we demonstrate that usage of distributed methods leads to high loss of assignment efficiency. In addition we suggest means of mitigating this advers effect.

Contents

Chapter 1

Introduction

Frequency planning is an important method to control co-channel interference in multi-cell communication systems. It is based on solving the frequency assignment problem (FAP) [1]. The FAP consists of a set of cells, where neighboring cells have certain (static) interference relationships and hence, should not be assigned the same frequencies (also referred to as channels in the following) for operation. The goal of the FAP is the assignment of a pre-specified number of frequencies to each cell while minimizing the total amount of frequencies needed. Mathematically, the FAP can be expressed as a graph coloring problem, by assigning each node one (or multiple) color(s) such that no two connected nodes have the same colors while trying to minimize the total number of colors used. A graph where the nodes represent the set of cells and the edges between the nodes represent their interference relationships is being used for this purpose.

This graph coloring problem is difficult; mathematically speaking, this problem belongs to the class of NP-hard problems. Finding the system optimum for practically relevant systems requires prohibitively long computation times even with modern computational equipment. Therefore two approaches are usually used:

- Suboptimal centralized algorithms which have a significantly reduced computational complexity while handling the full interference graph.
- Decentralized approaches, in which each node selects its frequency based only on partial knowledge of the interference graph. This allows for parallelization of the computation and leads to the most significant reduction of the computational time.

In the usually investigated wireless cellular networks with static frequency assignment both these approaches achieve remarkably good results in the sense of minimizing the number of frequencies necessary for assuring a given level of traffic, as compared to the real optimum. However, in the last decade an increased interest is observed in systems which are not “frequency-static” but change their operational frequency. Such systems do provide better immunity both against fading and interference. Such an approach is referred to as frequency-hopping. It is intuitively clear that if each cell applies frequency hopping the FAP approaches a new level of complexity. Thus, the issue of reducing the computational complexity becomes critical – and thus the promising decentralized approach described above is especially attractive.

In this paper we consider a special instant of such frequency hopping systems – the emerging IEEE 802.22 [2] standard for regional area networking. Its goal is to allow communication in temporarily unused TV bands (called secondary communication) but vacate the band if the owner of the band (called the primary user – PU) returns. In order to ensure an unimpaired operation of the PU, a used channel has to be sensed periodically by the 802.22 system. 802.22 features a hopping mode where a cell can hop over a set of channels. In this mode the channel to hop to is always sensed in parallel to the payload data transmission on the current channel, enabling non-disruptive communication. Although the hopping frequency is rather slow (in the order of seconds), there are evidently tough requirements on the computational complexity of the frequency assignment algorithms. On the other hand we believe that such optimization is feasible – in contrary to sytems with much faster hopping.

The remaining paper is structured as follows. In Section 2 we discuss related work regarding approaches for frequency hopping. In Section 3 we present our system model and formulate the problem statement. In Section 4 we present a precise centralized optimization approach to be used as reference for comparision, and introduce a candidate decentralized approach. Then, in Section 5 we investigate the performance of the distributed approach. Finally, we conclude the paper in Section 6 by discussion of options for improvement of the distributed approach.

Chapter 2

Related Work

The issue of “static” graph coloring for channel assignment is well documented in the literature and it has been frequently applied to cellular network planning. Standard references for this can be found in [3, 4]; for more in-depth studies, an excellent web page on the topic is maintained by Eisenblätter and Koster [1].

Frequency hopping has drawn significant research attention in the context of GSM cellular systems, Bluetooth and WLAN (among others). In GSM, frequency hopping is an optional mode to mitigate fast fading and co-channel interference. Once every TDMA frame (which has a duration of 4.17 milliseconds the transmit frequency for each terminal is changed according to a prespecified hopping sequence. The impact of this hopping sequence (also referred to as Mobile Allocation List – MAL) design in studied in [5]. The authors propose a scheme which generates frequency lists assuming the knowledge about the frequency lists of neighboring, i.e. interfering, base stations such that the interference between neighboring (hopping) cells is within some specified constraint. Further work on the assignment of frequency lists in GSM systems can be found in [6]. In contrast, [7] investigates *dynamic* frequency hopping in GSM and compares it to random hopping. The frequency hopping pattern of a mobile is adapted based on measurements made at the base station and the mobile. The recalculation is done after every TDMA frame. The paper studies several degrees of dynamic adaptations if the currently used frequency list is not satisfactory. However, the paper does not consider a jointly performed frequency list assignment over several cells.

Frequency hopping is also applied in Bluetooth systems for similar reasons (i.e. mitigating interference and fading). Hopping is performed about every 0.5 milliseconds and Bluetooth cells choose from several prespecified hopping sequences. The Bluetooth Special Interest Group (SIG) worked out an Adaptive Frequency Hopping (AFH) method for second generation Bluetooth devices to decrease the influence of so called fixed sources of interference (such as WLAN) on Bluetooth [8]. AFH allows Bluetooth to adapt to the environment by identifying fixed sources of interference and excluding them from the frequency hopping list. This process of re-mapping also involves reducing the number of channels to be used by Bluetooth. The Bluetooth specification requires a minimum set of at least twenty channels.

Chapter 3

System under Study and Problem Statement

IEEE 802.22 is an emerging standard for Wireless Regional Area Networks (WRANs) operating on license-exempt and non-interference basis in the spectrum allocated to TV broadcast services (between $47 - 910\,\mathrm{MHz}$). It aims at providing alternative broadband wireless Internet access in rural areas without creating harmful interference to licensed TV broadcasting. In this sense, it can be seen as a first cognitive radio (CR) system. 802.22 networks are organized in form of cells where a cell is envisioned to have a radius of several kilometers. An 802.22 cell consists of a base station (BS) and associated Customer Premise Equipments (CPEs). The BS has total control of all CPEs in its cell. Payload communication is set up by the BS scheduling the CPEs to sense the spectrum and collecting the sensing results. Based on these results it selects a payload communication channel for the cell. In order to avoid harmful interference with (re)appearing licensed users, this payload channel has to be sensed for PUs at least every 2 seconds. In order to avoid periodic interruptions of the payload communication for spectrum sensing, dynamic frequency hopping (DFH) has been proposed for 802.22 [2, 9].

3.1 Single Cell (Hopping) Operation

A secondary cell operates on one (at a time) arbitrary channel out of F_{tot} available ones. The maximum time period a secondary cell can interfere with a primary user is given by $t_{\max}$; consequently, the operating channel must be vacated at least after each $t_{\max}$ period (in order to be sensed and re-validated). Note that there are additional delays to be considered here, like the time needed for sensing the new candidate operating channel (t_{sens}), and the time needed for switching the operating channel of the cell (t_{switch}). The quiet time (t_{quiet}) is defined as $t_{\text{quiet}} = t_{\text{sens}} + t_{\text{switch}}$. We assume $t_{\max}$ to be a multiple integer of the quiet time ($t_{\max}/t_{\text{quiet}} = N_q$), and since $t_{\text{sens}} \gg t_{\text{switch}}$, we do not consider switching times in our investigations ($t_{\text{switch}} = 0$).

The BS can select from two basic modes of operation. The *non-hopping* mode uses static channel assignment where the payload communication is periodically interrupted (every $t_{\max}$) in order to perform sensing on that channel. The FAP in this case can be solved by applying one of the several existing graph coloring algorithms based on either global or local knowledge (hence, following either a centralized or a decentralized approach).

In the *hopping* mode the BS switches the cell to a new channel with periodicity of $t_{\max}$ seconds, even if its current channel is not used by a PU. The potential new working channel is previously sensed in parallel to the payload communication on the current channel. Hence, in the hopping mode the payload communication is interrupted only by a time span of t_{switch}, which we assume to be marginal. If no new channel is found to be available (due to PU or CR system activity), the base station switches to the non-hopping mode and immediately schedules a sensing period in order to check the current payload channel for PU activity.

3.2 Cellular Operation

We consider an area where a set of $|V|$ distinct CR cells are located. Depending on the distance between CPEs and BSs of several cells, it is possible that cells interfere with each other when operating on the same channel. We model this in form of an interference topology graph $G = (V, E)$ where $V = \{v_1, ..., v_n\}$ represents the set of CR cells and $(i, j) \in E$ if v_i and v_j are within each other's interference range (thus, the CR cells v_i and v_j cannot operate on the same channel at the same time). We assume that cells have means to discover the interference relationships within their neighborhood by exchanging control messages.

The presence of primary users is assumed to be static as well as the structure of the interference graph. Furthermore, we assume that if a PU appears, it affects all CR cells in the network.[1] Dynamically appearing and disappearing PUs as well as only locally visible PUs are subject to future work.

3.3 Problem Statement

Clearly, the hopping approach has the potential to support (almost) continuous service provision and thus the QoS needed for real-time applications. Additionally, the achievable throughput is much higher in the hopping mode ($5\,\%$ in 802.22 with $t_{\max} = 2\,s$ and $t_{\text{sens}} = 0.1\,s$). However, a network operating in hopping mode requires a larger amount of channels compared to the case where each cell operates in non-hopping mode. The channel usage is an important metric due to two reasons. The smaller the number of channels a CR network requires, the lower is the probability that a CR cell is operating on a channel which is reclaimed by a PU. In addition, the smaller the number of required channels, the more CR cells can operate on the same set of channels. Therefore, in this paper, we investigate the difference in terms of channel usage between non-hopping and hopping modes, i.e. the consequences of frequency hopping for the frequency assignment problem. In particular, we compare two different approaches, one with central and one distributed channel assignment. In the central approach a single node in the network has global knowledge and can compute the optimal frequency hopping assignments for all cells. In the distributed approach each cell decides on its own about the next frequency to be used only depending on the currently used frequencies of its neighbors.

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[1] In 802.22 the main class of PUs are TV broadcasters which have a much larger interference range compared to 802.22 cells. Additionally, they have a rather static behavior which does not change frequently over time.

Chapter 4

Generation of Hopping Sequences

In this chapter, we describe the process of generating the hopping sequences in a cluster of CR cells. We first describe the general approach followed, which was originally introduced in [9]. Subsequently, we introduce two algorithms for the hopping pattern generation. The first one is based on a central entity, which computes the hopping sequences for all cells based on global knowledge of the interference between cells (i.e., given the complete interference graph). In this approach the central entity has to distribute the generated hopping patterns to all cells in the network. The second approach is a decentralized algorithm, where each cell decides independently on its next used channel, only based on its local view of the interference and channel occupancy of neighboring cells.

The underlying idea of the hopping approach used is a *phase shifted* operation [9] of **neighboring** CR cells. The operation (i.e., the channel selection, as well as the jump to a new channel) of two neighboring cells is always time shifted by one quiet time (t_{quiet}). That means a cell has t_{quiet} time units to perform sensing, choose a working channel, jump to that channel, and announce the jump to its neighbors before the next cell within the neighborhood does the same. No two neighboring cells are allowed to perform sensing in the same slot or to hop to a new channel at the same time. Provided reliable communication between the cells this always ensures a collision free channel selection.

The principle of phase shifted operation limits the number of neighboring cells that can be supported. Since each cell needs to sense its working channel at least once per $t_{\max}$ and no two neighboring cells are allowed to perform sensing in the same slot, a maximum of $t_{\max}/t_{\text{quiet}} = N_q$ neighboring cells can be supported. To allow more than N_q neighboring cells, additional mechanisms to ensure a collision free operation would be needed. One possibility is to use distinct channel sets, i.e. up to N_q cells using the channel range between channel a and b and up to another N_q cells using the channel range between channel $b + 1$ and c resulting in a total number of $2N_q$ cells that can be supported. For our investigations, we assume that there are always enough slots, i.e. N_q is always bigger than the maximum number of neighbors in the network.

The general idea followed for the hopping sequence generation is based on an approach introduced in [9]. We refer to this approach as Revolver Hopping (RH). It is a straightforward hopping mechanism to support DFH in phase shifted operation while meeting the (regulatory) requirements on the maximum transmission time on a channel. After the maximum transmission time ($t_{\max}$) is over, a cell chooses a new working channel. Since each cell's

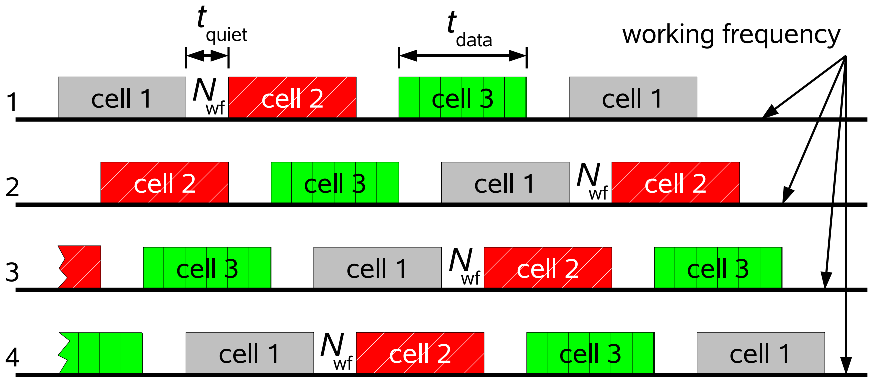

Figure 4.1: Revolver Hopping operation

operation period is time-shifted by one or multiple quiet time periods against the operation periods of all neighboring cells, there is always enough time to perform channel sensing on the next working channel before the cell performs the hop.

Thus, the $|V|$ cells coordinate their channel usage by hopping over a cluster of working frequencies. Figure 4.1 shows an example of the hopping pattern for 3 *neighboring* cells (i.e. all cells are mutually interfering with each other) using 4 channels. As can be seen in Figure 4.1 all cells consecutively visit each channel for the maximum allowed data transmission time ($t_{\max}$). In the figure, the sensing slots for the working channel (N_{wf}) of cell two are marked.

4.1 Centralized Approach – CH

In the hopping mode, the central entity needs to generate a channel assignment sequence per cell consisting of a *set of channels* and a schedule when to switch and to which channel. The channel assignment sequence has to be distributed to all cell in the network. In case of global knowledge, we suggest the following generation of hopping sequences. Initially, the central node computes the chromatic number χ_G[1] and the corresponding channel assignments of the network (of the graph G) solving the Linear Integer Program (LIP) in Eqs. (4.1-4.4)[2]:

$$\min \quad \chi_G = \sum_{\forall c \in C} y_c \tag{4.1}$$

$$\text{s. t.} \quad \sum_{\forall c \in C} x_{c,v} = 1 \qquad \forall\, v \in V \tag{4.2}$$

$$x_{c,v} + x_{c,w} \leq 1 \qquad \forall\, c \in C \wedge \forall (v,w) \in E \tag{4.3}$$

$$y_c \geq x_{c,v} \qquad \forall (c,v) \in C \times V \tag{4.4}$$

where $x_{c,v}$ is a binary assignment variable of color c and node v, constraint (4.2) assures that each node is assigned a color, and constraint (4.3) assures that neighboring nodes do

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[1] The chromatic number of a graph G is the minimum number of colors required to completely assign each node a color while ensuring non-interference of neighboring nodes.

[2] Note that the problem can also be approximated by heuristics. However, for our investigations we always used the system optimum.

not get the same color. Note that y_c is an indication variable denoting the usage of color c in the network at all (constraint 4.4). The network is represented by its interference graph $G = (V, E)$ as introduced in Section 3.2. C is the set of colors (channels) available.

Next, the central entity generates a fixed hopping sequence for each cell. The hopping sequence is generated based on the initial channel indices: First, all cells with channel index one switch to $\chi_G + 1$ simultaneously; after t_{quiet}, the cells with channel index two switch to channel index one etc., resulting in periodic channel hopping sequences for all cells. We refer to this approach as Centralized Hopping (CH).

The total number of channels required to operate the network is exactly $\chi_G + 1$ [9] due to the fact that hopping times are shifted – such that no two neighboring cells hop at the same time[3]. This is a lower bound for a hopping network regarding its channel requirement. However, notice that it is based on strong assumptions. The central entity has to collect the information regarding the complete interference graph, then it has to solve the above channel assignment problem and afterwards it has to reliably distribute the hopping assignments to all cells. We consider this approach mainly for comparison reasons in the following, rather than proposing it for practical usage.

4.2 Decentralized Approach – DHA

Because of scalability reasons, the above centralized approach is probably not applicable to larger network sizes. Therefore, we are interested in generating the hopping sequences in a distributed way based on local information only and quantifying the performance of this scheme.

As a basis, we took the Distributed Largest-First algorithm (DLF) [10], originally designed to solve static FAPs. This approach is known to perform near to optimal for static FAPs in practical problem instances. We modified DLF to handle the problem of generating the hopping sequences with the expectation to also perform well for this case.

The basic idea of DLF is the following: After discovering their cell neighbors, each node of the graph (i.e. each cell) collects information about the node degree (number of neighboring nodes) of its neighbors. The cells then choose their working channels in descending order of that node degree, i.e., the cell with the highest node degree selects its channel first. For equal node degrees a random number is used for tie breaking. A cell always chooses the lowest channel available and distributes its choice within the neighborhood. This method ensures that no two neighboring cells can get the same channel (as only one channel is chosen in a time).

Now consider the case of frequency hopping. We modify the DLF approach, referring to it as Decentralized Hopping Approach – DHA. Each cell performs the following steps: First, it initializes its neighbor list as described for the DLF. Then, all cells perform the priority selection procedure of DLF; the cells with the highest priority within their neighborhood choose a working channel (the lowest channel index available), communicate their choice to their neighbors, and start using the channel. After this, all cells with the second highest priority are allowed to choose their operating channel and so on. Up to this point, the channel

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[3] Note that $\chi_G + 1$ only holds as long as $N_q \geq \chi_G$, which is assumed for our investigations.

allocation is identical to that of the DLF algorithm. Note, however, that there is always a time shift of $t_{\text{sens}} + t_{\text{switch}}$ between the channel selection of two neighboring cells.

After using a channel for $t_{\max}$ seconds, a cell vacates the currently used channel and hops to the next available one with the lowest channel index. Note that the initial hopping order between the cells remains unchanged, since all cells use their channel for the same amount of time ($t_{\max}$) and due to the property of DLF that no two neighboring cells select their channels at the same time.

Although the order of the channel selection is periodic among the cells, it might happen that – depending on the choice of all other cells – the selection of the channels themselves results in an aperiodic channel hopping sequence. This effect is due to the "dynamic" choice of the next operating channel while system operation, and is a major difference to the centralized approach.

Chapter 5

Performance Comparison

In this chapter, we compare the number of channels used in case of the non-hopping and the hopping mode. For both cases we compare two approaches a central and a distributed one, as introduced above. As previously mentioned, the central approach should be regarded as a comparison case rather than as a practical approach.

5.1 Methodology

We randomly generated interference topology graph instances using Culbersohn's graph generator [11] on a 1 by 1 unit plane, with the number of nodes varying between $|V| = 10..50$. The nodes are connected (i.e., the cells are interfering) if their euclidian distance is smaller than or equal to d, where we vary this distance between $d = 0.35..0.6$. We have generated 80 random graph topologies for each of those $(|V|, d)$ pairs. We chose $t_{\max} = 10\,s$ and $t_{\text{sens}} = 0.1\,s$, resulting in $N_q = 100$. Since the maximum number of nodes is 50, for our investigation, it always holds that $N_q \geq \chi_G$[1].

In case of the centralized approach (CH), we transform the graphs into linear programs and compute the chromatic number using CPLEX [12]. Based on the result, the central node computes the channel assignment and distributes it within the whole network, as described previously. Since we do not assume interference from PUs, the assignment does not need to be changed over time and is always optimal.

In case of the distributed approach (DHA), we have implemented the DHA based on the above description. The initial channel usage is determined using the DLF. Afterwards, each node individually decides on which channel to jump based on the channel usage of its neighbors. In our implementation, neighbor nodes can reliably exchange their current channel usage in zero time. The simulation time is set to $1000\,s$.

We observe the total number of channels each graph instance requires over time. The maximum number of channels required over time is taken as performance metric for each graph. Afterwards, we average that number for both, the central and distributed approach over the $(|V|, d)$-graphs, for each $(|V|, d)$ pair.

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[1] In 802.22 $t_{\max}$ is two seconds, which would result in $N_q = 20$. Consequently, in some graph instances, there would not be enough sensing slots for all neighboring cells. To avoid cells operating in non-hopping mode (and, thus, investigate the theoretical potential of the hopping mode), we increased $t_{\max}$.

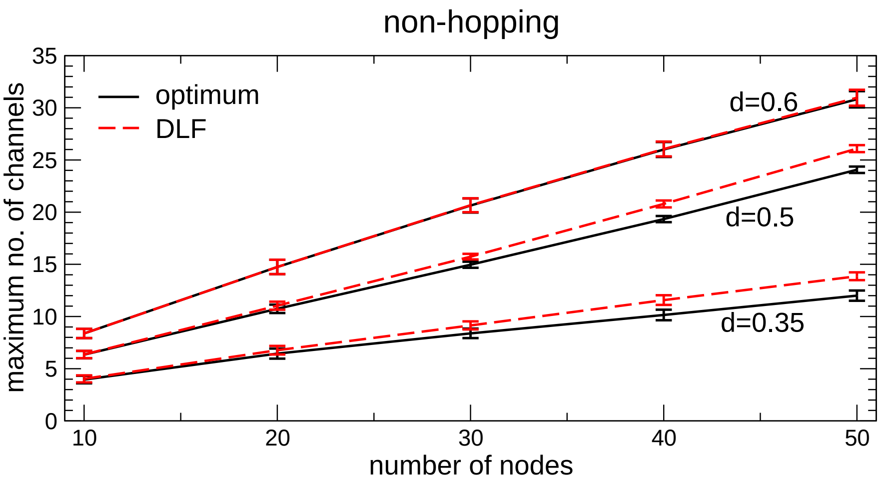


Figure 5.1: Average number of channels required for interference-free assignment for the non-hopping mode. We show the average results for the centralized and decentralized approach for a varying number of nodes per graph for two different interference ranges $d = 0.35$, $d = 0.5$, and $d = 0.6$.

5.2 Results

First, we present the results for frequency requirement in the non-hopping mode, i.e., the traditional frequency assignment problem. Afterwards, we study the same metric for the hopping mode, comparing the centralized, optimal solution to the DHA algorithm. We show that the performance difference between the centralized (CH) and distributed (DHA) approach increases significantly.

In Figure 5.1 we present results for three different interference distances ($d = 0.35$, $d = 0.5$, and $d = 0.6$) for the non-hopping mode. The key issue to observe from Figure 5.1 is that for the non-hopping mode, i.e. for traditional graph coloring, the performance difference between the centralized (optimal) and decentralized approach (DLF) is rather small. For $d = 0.6$ the difference is almost nill. This is in accordance with previous publications and holds for a wide set of graphs. Hence, for the non-hopping mode, the decentralized approach is much more preferable due to its easy and overhead-less operation.

In Figure 5.2 we show the number of channels required for operating the hopping network in the centralized or distributed fashion described above. Comparing Figure 5.1 & 5.2 we observe that for both – centralized and decentralized – algorithms, the hopping mode requires more channels. Whereas the difference between the centralized non-hopping and hopping approach is rather small ($\chi_G + 1$ compared to χ_G), the difference between the DHA and DLF is much larger (in other words, the cost of operating the network by the decentralized approach is much higher for the hopping mode). The DHA uses a lot more channels than the centralized hopping (CH) approach. This is rather surprising seeing the good results achieved by the DLF for the non-hopping mode.

This performance difference is further investigated in Figure 5.3. Here we present the probability mass function (PMF) of the number of required channels to operate a network in

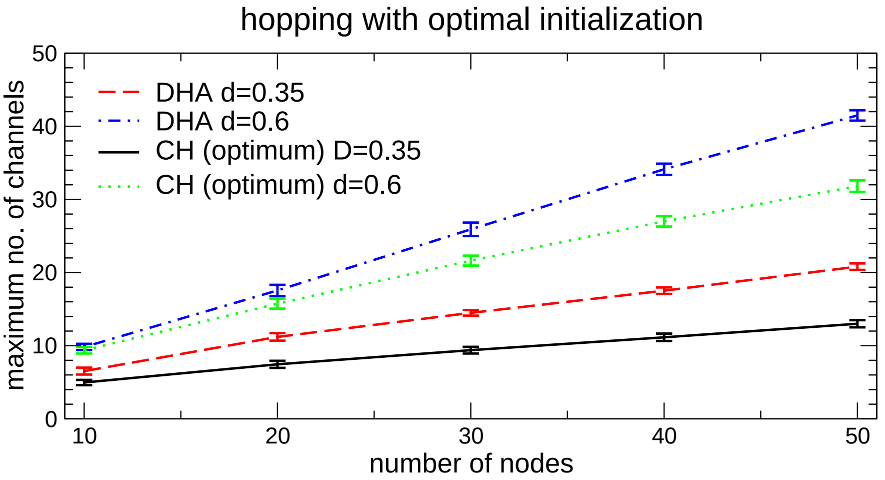
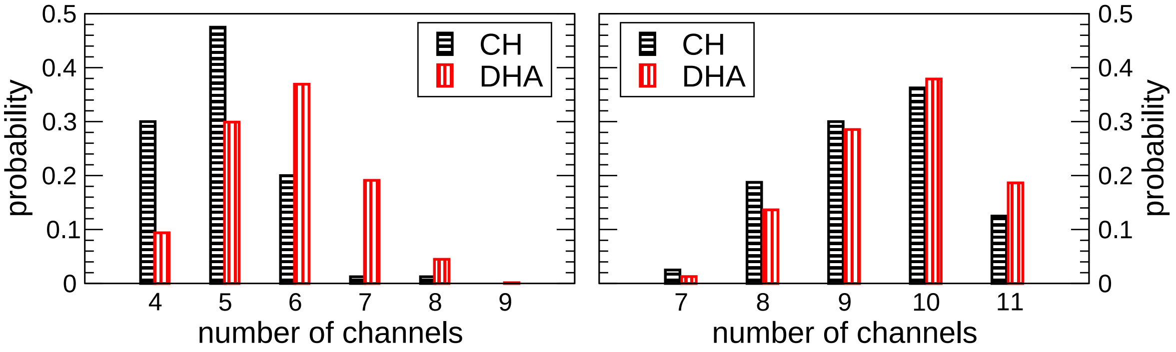
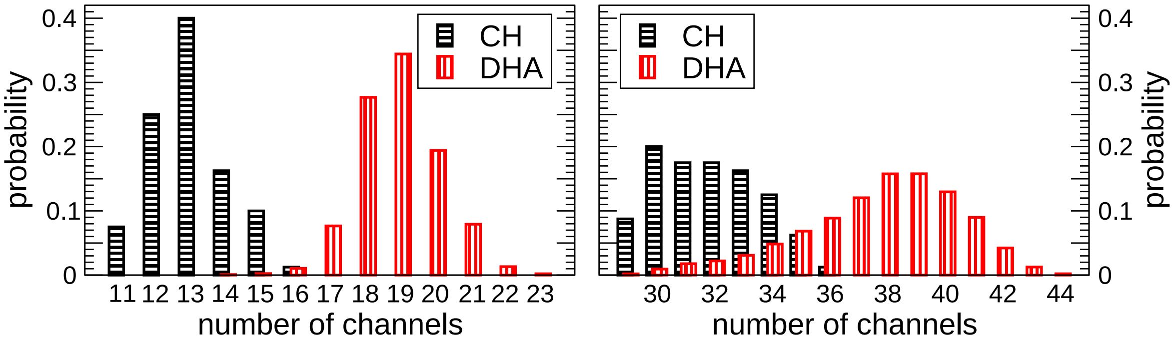


Figure 5.2: Average number of channels required for interference-free assignment in case of the hopping mode. We show the average results for the centralized and decentralized approach for a varying number of nodes per graph for two different interference ranges $d = 0.35$ and $d = 0.6$.

hopping mode for the CH and the DHA. The graphs show the probability that the network occupies a certain number of channels for many different graph instances. The PMFs for a small number of nodes (Figure 5.3(a)) of both approaches still have a big overlap. For $|V| = 50$, however, the PMFs for the CH and the DHA differ strongly. This is in accordance with the results shown in Figure 5.2, where for small $|V|$ the DHA can still achieve results comparable to the optimum. For an increasing $|V|$, however, also the difference between the DHA and the optimum increases.



(a) left: $|V| = 10$, $d = 0.35$, right: $|V| = 10$, $d = 0.6$



(b) left: $|V| = 50$, $d = 0.35$, right: $|V| = 50$, $d = 0.6$

Figure 5.3: PMF of the channel requirement for the centralized (CH) and decentralized (DHA) approaches in case of the hopping mode.

Chapter 6

Conclusions

We have discussed the impact of frequency hopping on the required number of channels for cellular networks motivated by the current activities in the 802.22 working group. We have introduced the decentralized hopping approach (DHA) which supports frequency hopping for 802.22-like cellular networks. It has been shown that, unlike in non-hopping mode, this decentralized hopping approach performs much worse than the centralized one in terms of the number of required channels. This is important as the total number of required channels determines the potential impact of cognitive radio (i.e. secondary) interference on primary users. The centralized hopping (CH) algorithm, however, needs only a moderate increase of required channels compared to the non-hopping centralized one.

Despite the negative result presented, we are still convinced that frequency hopping in CR networks can be realized in a distributed fashion, achieving comparable results as the optimal assignment. There are several items we plan to investigate in the future:

- What is the influence of the specific hopping approach (the Revolver Hopping (RH) approach) and its decentralized realization (DHA) on the presented results? Are there other hopping approaches that can achieve better results, given the same amount of neighborhood information ?

- What is the influence of the amount of neighborhood information in each node? Can the performance of the DHA be improved by knowing the channel usage of all n-hop neighbors? Our preliminary results motivate the introduction of cooperation between hopping cells (forming for example communities): We will study the performance impact when each such community has regional information about its vicinity and the corresponding overhead required to keep this information up to date.

- Finally, we are interested in investigating the impact of primary user dynamics on the performance results of the centralized and decentralized approach.

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