Technical University Berlin

Telecommunication Networks Group

Dynamic Mechanisms in OFDM Wireless Systems: A Survey on Mathematical and System Engineering Contributions.[a]

James Gross, Mathias Bohge

{gross,bohge}@tkn.tu-berlin.de

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Abstract

In this paper we review contributions regarding dynamic, i.e., channel-state adaptive schemes for OFDM systems. As the wireless broadband channel is frequency-selective, the OFDM transmission scheme provides an efficient technique to overcome this impairment. However, the OFDM transmission scheme provides further advantages. Various system metrics can be improved by periodically reassigning transmit power and sub-carriers to terminals depending on the current sub-carrier attenuations. We review such schemes regarding point-to-point connections and point-to-multi-point connections. In both cases, a lot of scientific effort has been spent on modeling and solving optimization problems. In addition to these mathematical studies, we also review system-related contributions considering the required control information or the feed-back of channel knowledge from the receiver to the transmitter. Several novel results regarding the complexity of two different optimization approaches and various comparisons of different sub-optimal algorithms complete the paper.

Contents

Chapter 1

Introduction

As ever higher data rates are to be conveyed by wireless communication devices, the frequency-selective nature of wireless channels gets a limiting factor for the systems performance. This has lead to the development of multiple transmission schemes, one of them being **O**rthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) . Although the basic principle of OFDM has been known for quite a while [1–4], the application to mass-market communication systems just started a couple years ago.

The basic principle of OFDM is parallelization: Instead of transmitting symbols sequentially over the communication channel, the channel is split into many sub-channels and the digital symbols are transmitted in parallel over these sub-channels. This increases the symbol duration per sub-channel as the sub-channels bandwidth decreases. A frequency-selective attenuation of the *single* overall channel becomes a flat attenuation per sub-channel, improving the performance in frequency-selective channels significantly. Hence, OFDM systems are excellent systems for the application to wireless broadband channels.

This property of OFDM systems has lead to the specification of various systems based on OFDM. Modern digital audio [5] and video [6] broadcasting systems rely on OFDM. Some well known High speed wireless **L**ocal **A**rea **N**etwork (*LAN*) (e.g. IEEE 802.11a/g [7]) standards are based on OFDM, as well as other wireless network standards such as IEEE 802.16 [8]. However, OFDM has also been applied to wired frequency-selective channels, as in the case of **D**igital **S**ubscriber **L**ine (*DSL*) systems for twisted-pair cables [9]. Due to this recent popularity of the OFDM transmission scheme, it is also considered as candidate for high-rate extensions to third generation communication systems as well as for fourth generation mobile communication systems.

As OFDM systems provide excellent physical layer properties, they also offer interesting opportunities regarding link layer aspects. Due to the fine granularity of the sub-channels, resource requirements of terminals can be served without much over-provisioning of bandwidth. This leads to the idea of adapting the bandwidth to the rate requirements of terminals. In addition, due to the frequency-selective attenuation of the wireless channel, the modulation type and the transmit power per sub-channel can be adapted in order to increase the spectral efficiency. Moreover, in a multi-user scenario, the spatial diversity of the attenuation offers the opportunity to assign *dynamically* different sets of sub-carriers to different terminals. Within the last ten years a lot of scientific effort has been spent on such dynamic schemes in OFDM systems, highlighting the potential gain stemming from them. However, such schemes demand also challenging requirements. In this survey, we provide an overview of the research in this area. Major research results are summarized while also open issues are identified for future investigations. Thus, the survey serves the interested engineer as overview and orientation while it also encourages scientists to further investigate certain areas. Some own contributions regarding the performance of optimal and suboptimal approaches as well as a complexity analysis of a frequently addressed optimization problem complete the paper.

The paper is structured as follows. After a brief introduction to the OFDM transmission scheme, we first focus on dynamic mechanisms for point-to-point connections (Section 3). In this case, it is possible to vary the transmit power as well as the amount of data transmitted on each sub-carrier. We discuss in this section various modeling approaches as well as solution strategies for the resulting optimization problems. Next we focus on point-to-multi-point connections (Section 4). In the case the resutling optimization problems are more complex, as is shown in the appendix. However, performance studies show the large potential of this approach, even if only sub-optimal schemes can be employed. In addition, the issue of the control overhead is discussed in Section 4. Finally, we conclude the paper in Section 5.

Chapter 2

The Orthogonal Frequency Division Multiplexing (*OFDM*) transmission scheme

In this section we briefly review the OFDM transmission scheme. The interested reader may find in depth introductions to the OFDM scheme in [10–12]. n OFDM systems the available bandwidth B [Hz] is split into N *sub-carriers*, also referred to as *sub-channels*. Instead of transmitting digital symbols sequentially through *one* channel (of bandwidth B [Hz]), the bit stream is split into N parallel streams (cf. Figure 2.1). Then bits from each stream are converted into digital symbols and transmitted in *parallel*. The parallel transmission has important consequences for the symbol length. While a (sequentially transmitting) equalized **S**ingle**c**arrier **M**odulation (*SCM*) system has a symbol length of T_s, the symbol length of an equivalent[1] OFDM system is N times longer (due to the fact that during each symbol duration N symbols are transmitted in parallel).

The exact generation of the OFDM symbol works as follows. After splitting the transmit bit stream into parallel data streams, groups of bits of each stream are mapped to the frequency-domain representations of the corresponding digital symbols of some modulation alphabet. Each sub-carrier might be modulated differently. These N symbol representations are passed to an inverse, Fast Fourier transformation (the IFFT), which generates a *time sequence* of N values. This time sequence represents one OFDM symbol. It relates to the duration of an OFDM symbol. The sequence is then transmitted at a certain center frequency f_c with a certain transmit power P_{tx}. At the receiver the signal is passed to a Fast Fourier transformation. After applying this transformation, the frequency domain representations of the digital symbols for each sub-carrier are obtained. They are converted individually into bits, which ultimately yields the bit groups of each stream.

The increase of the symbol duration is the most significant advantage of OFDM systems when facing frequency-selective channels (for example, broadband wireless channels). Due to the "echo" caused by multi-path signal propagation, different "path copies" of a transmitted digital symbol arrive at the receiver with a different delay. If this "echo", technically referred to as delay spread $\Delta\sigma$ of the channel, is rather large compared to the symbol duration, sequentially transmitted symbols might interfere at the receiver (the fastest copies of the second symbol will interfere with the slowest copies of the first symbol). This effect is called **I**nter**s**ymbol **I**nterference (*ISI*) . As the symbol duration is

[1]Equivalent refers in this context to the overall number of symbols that are transmitted per time unit.

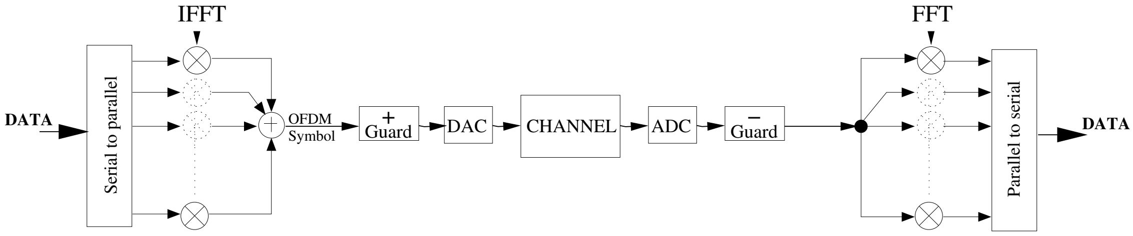


Figure 2.1: A simple OFDM transmission sketch.

increased by the factor N in OFDM systems, ISI can be mitigated by an appropriate choice of N.

As future communication systems are generally expected to convey higher and higher data rates and thus require larger bandwidths, ISI becomes more relevant. OFDM systems can utilize larger system bandwidths by increasing the number of sub-carriers, which increases the symbol duration, while still maintaining a high overall symbol rate. Hence, OFDM systems are excellent transmission systems for frequency-selective channels. By dividing the system bandwidth into N sub-carriers, per sub-carrier the attenuation of the channel becomes flat (if the system has been designed appropriately, i.e. the resulting symbol duration is sufficiently larger than the delay spread). In order to mitigate the effect of ISI completely, a cyclic extension of the OFDM symbol time sequence is added. This extension is usually larger than the delay spread and is called the guard period. It is discarded at the receiver prior to applying the FFT and removes any interference with previous OFDM symbols.

Chapter 3

Dynamic Schemes for Point-to-Point Connections

In this section we review results regarding the adaption of transmit power and modulation types for OFDM systems. In general, these adaption schemes exploit the frequency-selective nature of wireless channels. The frequency-selectivity of wireless channels stems from the multi-path propagation. Per sub-carrier the attenuation is usually flat (otherwise ISI occurs). However, different sub-carriers experience in general different attenuations, at least if the frequency difference between them is rather large. Within a certain bandwidth spacing the attenuation of sub-carriers is correlated. This bandwidth spacing is called the *coherence bandwidth* and it is an important criteria for evaluating a transmission channel. The coherence bandwidth depends on the delay spread, a mathematical definition can be found in [13, 14]. Within the coherence bandwidth the correlation of the attenuation is strong, leading to quite similar values of the attenuation.

Let us assume this frequency-selective behavior to stay constant for a reasonable amount of time. A transmitter has data to be sent to a receiver. Does it make sense for the transmitter to "adapt"[1] in a certain way to the frequency-selective attenuation of the channel in order to transmit the data "better" (i.e. faster, more reliable etc.)?

From information theory the "water-pouring" theorem [15] states an important answer to this question (cf. Figure 3.2). Given the transfer function of any channel, it provides the capacity[2] of this channel. The capacity is achieved by *adapting* the transmit power to the transfer function. Roughly speaking, given a limited transmit power more power is applied to frequency areas with a lower attenuation compared to the other frequencies[3]. Assuming a fixed average attenuation of the channel and a fixed bandwidth, the capacity of the channel increases the more *diverse* the transfer function of the channel is (i.e. the higher the variance of the transfer function is [13]). The lowest capacity of a channel results from a flat transfer function.

Apart from the fact that the *optimal* power distribution is computationally somewhat difficult to

[1] Throughout this chapter we refer to schemes which adapt to any channel variations as *dynamic*. In contrast, schemes which do not adapt to channel variations are referred to as *static*.

[2] In information theory the capacity is defined as the maximum bit rate at which data can be transmitted such that the bit error probability is arbitrarily small.

[3] Given the transfer function, the optimal power distribution is similar to inverting the transfer function and pouring a liquid (water), i.e. power, into the shape. Hence, the scheme was termed "water-pouring" (cf. Figure 3.2). Sometimes this scheme is also referred to as "water-filling" [13].

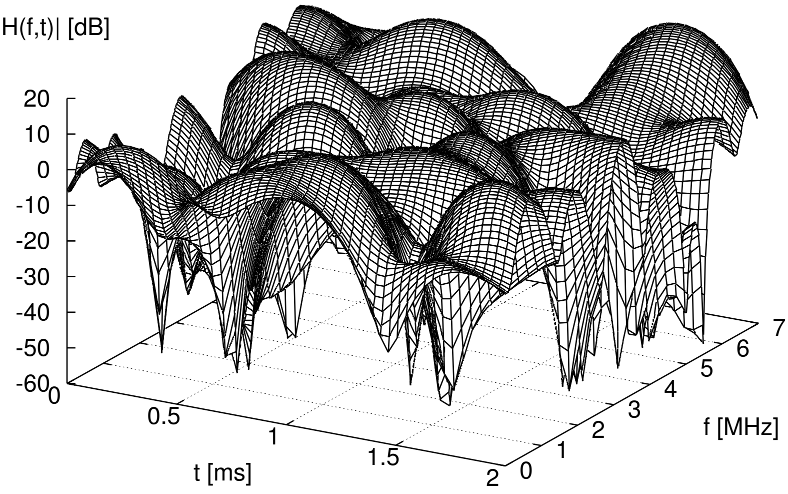


Figure 3.1: Time and frequency selective attenuation of a broadband wireless channels

obtain, the "water-pouring" result can not be applied directly in OFDM systems due to two reasons. First, the result holds only for a continuous transfer function whereas OFDM systems are characterized by a sample version of the transfer function (in other words, the "water-pouring" theorem assumes that the power can be adapted for infinitely small sub-carriers). Second, the metric considered is capacity. In particular, using capacity provides a continuous relationship between spent transmit power and received amount of bits per second. Applied communication systems do not have this property, they only have a limited set of different modulation types. These types provide only discrete steps of data rate.

The application of the "water-pouring" result to OFDM wireless systems calls for a discrete version of it, considered as *adaptive loading algorithms* [16]. Frequently, the terms bit- or power-loading algorithms are also mentioned. Bit-loading algorithms adapt the number of bits transmitted per sub-carrier according to the sub-carrier states. Correspondingly, power-loading algorithms adapt the transmit power. However, often the number of bits is adapted together with the transmit power. Thus, such schemes are simply referred to as adaptive loading algorithms in the following.

Given a certain power budget $P_{\max}$, the number of sub-carriers and a relationship between the data rate, the error probability and the SNR per sub-carrier (which results from the transmit power and the attenuation), a loading algorithm generates a power and/or modulation allocation for each sub-carrier. Two major objectives are considered [16]: Maximizing the data rate for a given power budget and a target bit error probability (called the bit rate maximization problem), or minimizing the transmit power for a certain given rate and a target bit error probability (called the margin maximization problem). The principle challenges regarding loading algorithms are twofold: How to generate the optimal allocation or at least a considerably "good" solution, and how to implement such a scheme in a real system. In the following two sections we separately highlight contributions to these two issues.

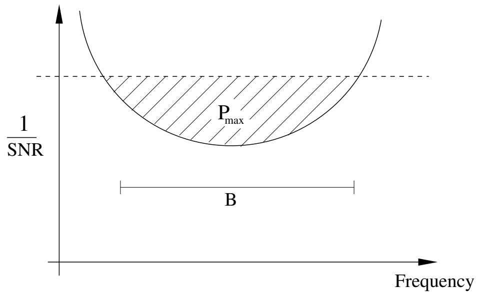


Figure 3.2: Principle of information theory's "water-pouring" approach.

3.1 The Generation of Optimal Allocations

Initially, let us consider a system with a discrete number of sub-carriers. However, assume a continuous relationship between transmit power and achievable number of transmitted bits per symbol (as in the capacity formula 3.2: an infinitesimal increase of power leads to an infinitesimal increase of bit rate) [17].

Consider $n = 1, 2, \dots N$ sub-carriers to divide the given system bandwidth B (each sub-carrier having a bandwidth of $\Delta f = B/N$). Per sub-carrier n and time point t the transmit power can be varied, denoted by $p_n^{(t)}$. All current power allocations yield the power vector $\vec{P}^{(t)} = \left(p_1^{(t)}, \dots, p_N^{(t)}\right)$. The attenuation for each sub-carrier n is denoted by $h_n^{(t)}$. This yields the SNR per sub-carrier, assuming a noise power of σ^2 per sub-carrier:

$$v_n^{(t)} = \frac{p_n^{(t)} \cdot \left(h_n^{(t)}\right)^2}{\sigma^2} \quad . \tag{3.1}$$

For a given SNR of a sub-carrier the capacity can be computed as:

$$r_n^{(t)} = \Delta f \cdot \log_2 \left(1 + v_n^{(t)}\right) \quad . \tag{3.2}$$

The capacity of the entire system is obtained by the sum of the capacities of each sub-carrier. Given the power budget $P_{\max}$, an optimization problem can be formulated, maximizing the capacity by distributing the transmit power (also referred to as bit rate maximization problem):

$$\max \qquad \Delta f \cdot \sum_{\forall n} \log_2 \left(1 + \frac{p_n^{(t)} \cdot \left(h_n^{(t)}\right)^2}{\sigma^2}\right) \qquad \text{s. t.} \qquad \sum_{\forall n} p_n^{(t)} \leq P_{\max} \quad .$$

(Finite Tones Water-Pouring)

Problem (Finite Tones Water-Pouring) is a non-linear, continuous optimization problem. It can be solved analytically by applying the technique of Lagrangian multipliers [18]. This technique yields after some some standard transformations Equation 3.3 as solution for the optimal transmit power per sub-carrier.

$$p_{n,\text{opt}}^{(t)} = \frac{1}{N}\left(\sum_{\forall i} \frac{\sigma^2}{\left(h_i^{(t)}\right)^2} + P_{\max}\right) - \frac{\sigma^2}{\left(h_n^{(t)}\right)^2} \tag{3.3}$$

This analytical expression represents the intuitive result of the water-pouring solution: The lower the relative attenuation of some sub-carrier is (compared to all other attenuations), the more transmit power this sub-carrier will receive. However, the result of Equation 3.3 has some disadvantages. The analytical expression may yield *negative power allocations* for some sub-carriers. In this case a realizable optimal power allocation for the system is generated by discarding all sub-carriers with a negative power allocation. Then, the optimal allocation is recomputed for the remaining sub-carriers. Eventually, a valid power allocation is obtained. In the worst case the complete power allocation has to be redone $N-1$ times until an optimal allocation is obtained.

If only a fixed amount of modulation types is available, a different approach has to be taken. Assume M different modulation/coding combinations to be available. Denote the relation between SNR, error probability and achieved bit rate by $F(\mathrm{SNR}, p_{\mathrm{err}})$. Using this function, for a given target bit error rate the required SNR can be calculated in advance for each modulation/coding combination. Thus, each sub-carrier can only be allocated one out of $M+1$ different power levels[4], if the current attenuation of each sub-carrier is given. Then, the bit rate maximization problem (Finite Tones Water-Pouring) becomes an integer programming optimization problem [19]. This can be formulated as:

$$\max_{\vec{P}^{(t)}} \quad \sum_{\forall n} F\left(\frac{p_n^{(t)} \cdot \left(h_n^{(t)}\right)^2}{\sigma^2}, p_{\mathrm{err}}\right) \qquad \text{(Bit Rate Maximization)}$$
$$\text{s. t.} \quad \sum_{\forall n} p_n^{(t)} \leq P_{\max}$$

In general, integer programming problems are difficult ones. Although the amount of possible solutions is finite (each sub-carrier might be allocated one of $M+1$ power levels, therefore there are $(M+1)^N$ possible power allocations), finding the optimal solution remains a difficult task, possibly requiring a brute force enumeration of all feasible solutions and comparing their achieved performance. In contrast, *efficient* algorithms are characterized by never requiring this enumeration.
n iterative approach to obtain the optimal bit- and power allocation to each sub-carrier has been patented by Hughes-Hartogs [20]. The principle of this algorithm is quite simple (cf. Algorithm 1): For each sub-carrier, calculate the amount of power required to transmit data with the lowest modulation/coding combination. Then, the sub-carrier which requires the least amount of power is selected, the amount of power is allocated to it and the required additional power for applying the next higher modulation/coding combination is calculated for this sub-carrier (while the total amount of available power is decreased by the allocated amount). The algorithm terminates if no more transmit power is available. It determines for a discrete amount of modulation/coding combinations the optimal power allocation with respect to the target bit error probability while maximizing the data rate[5]. Although the Hughes-Hartoges algorithm does note enumerate all feasible solutions, the required amount of steps is quite high. For example, assume the M modulation steps to differ by one bit. Then, for transmitting a total of 1000 bits the algorithm will have to perform 1000 iterations.

Therefore, faster schemes reaching the optimal or near-optimal power allocation have been of interest. Primarily they were discussed in the context of OFDM applied to **D**igital **S**ubscriber **L**ine

[4] A sub-carrier may also be allocated no transmit power at all. Therefore, there are $M+1$ different power levels.

[5] The exact same scheme can also be used to determine the optimal power allocation in order to minimize the transmit power subject to a rate constraint. In this case the algorithm simply runs until the target data rate is reached.

```
Algorithm: Hughes-Hartogs
Result : Optimal Sub-Carrier Power Allocation

Given an overall maximum power value P_max and the set of the current attenuation values
for each sub-carrier. Then, a transmit power matrix of values p^(t)_{m,n} is calculated in advance,
holding the required transmit power to convey m bits on sub-carrier n. The optimal power
allocation can be obtained using the following procedure:

Calculate the "incremental power matrix", consisting of values Δp^(t)_{m,n} = p^(t)_{m,n} − p^(t)_{m−1,n}
Set P_tot = 0
while P_tot ≤ P_max do
    Search row 1 of the incremental power matrix for the smallest Δp^(t)_{1,n}
    if (Δp^(t)_{1,n} is the smallest) then
        Assign one additional bit to sub-carrier n.
        Increment P_tot = P_tot + Δp^(t)_{1,n}
        Update column n of the incremental power matrix by Δp^(t)_{i,n} = Δp^(t)_{i+1,n} (move all
        terms of this column up by one position)
    end
end
```

Algorithm 1: The Hughes-Hartogs Bit-Loading Algorithm.

(*DSL*) systems. However, for the discussion in the context of wireless systems the allocation algorithms are equivalently relevant.

Chow et al. [21] presented a faster loading algorithm in order to minimize the transmit power while maintaining a required data rate. They propose to start with an *equal* power distribution and then alter this distribution in order to reach the required rate. After obtaining the bit rate per sub-carrier with an equal power distribution, they iteratively increase or decrease the transmit power margin per sub-carrier, depending on the difference between currently achieved data rate and target rate (for example, if the total rate is larger than the target, the system can accept a higher noise margin while still providing the target rate, therefore the transmit power for all sub-carriers can be lowered by a certain *factor* for all sub-carriers). This refinement process is performed for a certain number of times. Compared to the Hughes-Hartogs algorithm, this proposal determines an allocation much faster. However, it provides a suboptimal result (the difference is rather small in the DSL context, as presented by the authors). Fischer et al. presented in [22] an extension to this work with an algorithm yielding a minimum bit error probability while achieving a target data rate. Further schemes were presented in [23–26].

In contrast to the presented ideas so far, one can also consider pure bit-loading (fixed transmit power per sub-carrier). As the attenuation values differ strongly, the resulting SNR per sub-carrier varies, too. These varying SNR's motivate the idea to adapt the modulation types solely, referred to as *adaptive modulation* [11]. Given a certain target bit error probability, for each modulation type an SNR range can be obtained for which it is applied. Then, assuming the knowledge of the sub-carrier attenuations, for each sub-carrier the modulation type is simply adapted according to the SNR ranges.

An excellent, in depth discussion of adaptive modulation for multi-carrier systems is given in [11].

However, another concept is to vary the transmit power solely (pure power-loading) while considering an OFDM system with a single modulation type. This has been presented by Hunziker et al. in [27]. As the throughput is fixed (for each OFDM symbol the same number of bits is transmitted), the objective is to minimize the bit error probability while subject to a total transmit power budget. Since the transmit power is varied whereas only one modulation type is available, the respective optimization problem becomes again non-linear, continuous problem. Using the Lagrange multiplier technique, they obtain an expression for the optimal power allocation in case of perfect channel knowledge. A related work for power and modulation adaption has been presented by Mutti et al in [28].

3.2 System Aspects of Dyn. Schemes for Point-to-Point Connections

3.2.1 Performance Results

After discussing the computational effort required to optimally allocate power and bits to sub-carriers, the most important question relates to the performance gain that can be achieved by doing so. Comparing the achieved data rate in case of the bit rate maximization problem to a linear equalized SCM system, Wilink et al. find in [29] a 5 dB gain (target bit error probability of 10^{-5}) for a wide range of total transmit power. Compared to the channels capacity, the data rate achieved by adaptive loading is 8 dB lower.

In [30] Czylwik compares the performance of an SCM system with frequency equalization to the performance of an OFDM system with fixed and adaptive modulation. For different wireless channels Czylwik finds an improvement of around 2 dB when switching from the SCM system to the fixed OFDM system (both applying the same modulation type), and a further improvement of around 4 dB when switching from the fixed OFDM system to the OFDM system with adaptive modulation. In case of no line-of-sight, the performance gains are much larger, around 8 dB for switching from the SCM system to fixed OFDM and around 5 dB when switching from the fixed to the adaptive OFDM. Similar results advocating adaptive modulation were found by Rohling et al. in [31]. In a further study [32], Czylwik investigates the performance difference (in terms of bit error probabilities) between adaptive modulation (with fixed power allocation) and adaptive loading (variable power and bit allocation). For all considered channels (two different ones, relying on measurements) fixed OFDM is significantly outperformed (by about 5 dB by OFDM with adaptive modulation. However, the difference between adaptive modulation and adaptive loading is rather small, around 1 dB. This indicates that the computational expense related to adapting the bit *and* power allocation is not worth the performance gain achieved, at least for these channel characteristics studied. Similar results were found by Barreto et al. in [33] for an IEEE 802.11a-like system applying adaptive modulation and adaptive loading[6]. In [28] the authors find that the biggest advantage for a coded, interleaved system is obtained by performing adaptive modulation and not by adaptive loading of bits and power (adaptive modulation achieves at a 3 dB gain versus a static system, while additional power adaption only adds 1 dB more). In [27] the same authors show that soft decision decoding yields a much larger performance gain than adaptive power allocation.

[6]In IEEE 802.11a a technique called *link adaption* is performed. In link adaption the modulation type is varied as in adaptive modulation. However, all sub-carriers employ the same modulation type. Sub-carriers are not modulated individually.

However, it should be mentioned that both, bit-*and* power-loading, are employed in the context of DSL systems. The reason for this is that in the context of DSL the sub-carrier attenuations are much more diverse than in typical wireless transmission systems. In [25] it is mentioned that the attenuation over twisted pair loops might vary by 60 dB and more, which is a much higher variation than in wireless systems (as a rule of thumb a 10 dB fade has a probability of 10^{-1}, a 20 dB fade has a probability of 10^{-2} and so on [14]). This indicates that the diversity of the transfer function plays a major role.

3.2.2 Channel Knowledge Accuracy

So far perfect channel knowledge has been assumed at the transmitter as well as at the receiver. In real systems any channel knowledge is erroneous to some extent. In this context two problems arise. How does the transmitter obtain the receiver's channel states and how does an adaption scheme perform in time-varying channels. The first issue is strongly related to the duplex scheme. In **T**ime **D**ivision **D**uplex (*TDD*) the channel can be assumed to be reciprocal which simplifies the acquisition of the channel knowledge greatly. In **F**requency **D**ivision **D**uplex (*FDD*) the channel information has to be reported back somehow [34]. The second issue relates to the performance loss due to realistic channel knowledge at the receiver. For example, the receiver estimates the channel at the beginning of a transmit phase based on pilots. During the transmission, the current channel states might differ more and more from the estimate as the wireless channel is time-selective. In [30] a comparison is presented between the system performance in the case of channel estimation (based on the pilot symbols) and the performance with perfect channel knowledge. Several different mobile velocities ($0 - 10$ m/s) are considered. The results indicate that as long as the mobile velocity is rather small (about 2 m/s) the performance loss is small. However, for a velocity of 10 m/s the performance loss is quite significant. Note that the performance with channel estimation at high velocities is still considerably better than the one achieved by SCM systems with equalization (not considering ßtechniques) at these velocities. Similar results have been presented in [33] for adaptive modulation in IEEE 802.11a systems. Hunziker et al. develop in [27] a power loading scheme assuming an outdated channel knowledge at the receiver. With this improved allocation scheme they demonstrate a small performance gain (with respect to the bit error probability at a fixed data rate) compared to equal power allocation.

3.2.3 Signaling Overhead

At least for bit-loading schemes the receiver has to be informed of the chosen modulation types for the next allocation cycle in order to decode the sub-carrier symbols correctly. Thus, the transmitter has to inform the receiver. Obviously, this requires system resources. However, this performance loss has never been quantized so far. Some investigations though consider countermeasures (without evaluating the performance loss as such). Hunziker et al. propose in [27] to only vary the transmit power and apply only one modulation type (power-loading). Nguyen et al. [35] study the compression gain that can be obtained when applying loss-less compression schemes (run length codes in combination with universal variable length codes) to the signaling information. They find quite good compression gains of about 0.7. Alternatively, one can consider to exploit the correlation in frequency of sub-carrier attenuations by grouping various sub-carriers together to a sub-band and then allocate power and modulation types adaptively to these sub-bands. This reduces the signaling overhead since only for each group of sub-bands the new modulation type will have to be indicated (instead of for each sub-carrier). Lei et al. suggest such a scheme in [36]. However, their results indicate that the performance loss due to this method is only small, if the groups are relatively small. For a 20 MHz system with 512 sub-carries and considering indoor (delay spread at about 0.8 μs) and outdoor (delay spread at about 5 μs) propagation environments, a grouping of already eight sub-carriers leads to a severe performance decrease such that the application of a fixed OFDM system performs at the same level.

Chapter 4

Dynamic Schemes for Point-to-Multi-Point Connections

In this section we review results regarding the application of dynamic mechanisms in point-to-multi-point scenarios, i.e. the down-link transmission direction. First, we give an introduction to the nature of the problems arising in this context. Then, we discuss mathematical contributions to these problems.

The basic set up of a multi-user down-link transmission is shown in Figure 4.1. In such a scenario, the given system resources (power, bandwidth, time) are shared by several terminals. For example, in today's IEEE 802.11a/g [7] system, the system resources are shared according to **T**ime **D**ivision **M**ultiple **A**ccess (*TDMA*) . Thus, time is slotted and each terminal j is allowed to exclusively use all sub-carriers during some acquired time-slot. During each TDMA slot the connection becomes a point-to-point connection, allowing the application of dynamic schemes presented in the previous section.

However, another opportunity arises from an effect referred to as *multi-user* diversity. As several terminals are located in the cell, sub-carriers are likely to have completely different attenuations for several terminals. In other words, the multi-user communication scenario is characterized by a spatial selectivity of the sub-carriers, also referred to as multi-user diversity [37]. The reason for the spatial selectivity is the fact that the fading process (as well as path loss and shadowing) is statistically independent for different terminals, as long as their receive antennas are separated considerably[1]. Hence, dynamic **O**rthogonal **F**requency **D**ivision **M**ultiple **A**ccess (*OFDMA*) schemes appear to be promising in order to enhance the performance in multi-user scenarios.

The general system set up for such a dynamic scheme is quite similar to dynamic mechanisms for point-to-point connections. Initially, assume the attenuations of sub-carriers to be stable for a certain time span (coherence time). The access point has the knowledge of the current attenuations and a dynamic algorithm at the access point generates *disjunctive sets of sub-carriers* assigned to each terminal, possibly including individual modulation types and different power allocations per sub-carrier. Then the access point informs each terminal of its next assignment set and starts the payload data transmission. The sets are valid for the length of one down-link phase.

[1]There is no definition for the minimum spacing but a spacing of one wavelength is assumed to be sufficient.

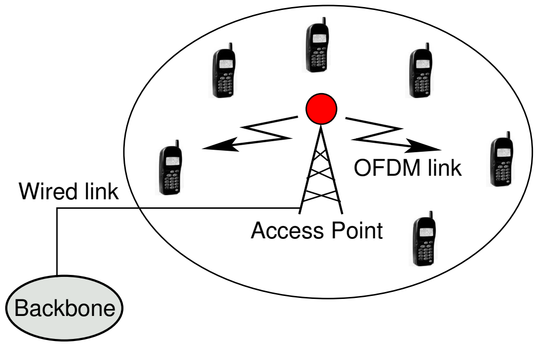


Figure 4.1: A cellular point-to-multi-point OFDM scenario, consisting of one transmitter (base-station) and several receivers (terminals).

4.1 Optimization Problems in Dynamic OFDMA Systems

Consider the system model that was introduced in Section 3 as basis for the optimization problem (Bit Rate Maximization), but recall that this time J terminals are present in the cell. The system is characterized by N sub-carriers and M different available modulation/coding combinations. Again, $F\left(SNR, p_{\mathrm{err}}\right)$ denotes the general relationship between the SNR per sub-carrier (which depends on the transmit power, the attenuation, and the noise power) and the conveyable amount of bits at a target bit error probability. As each user j usually experiences a different attenuation for each sub-carrier n, a suitable system description is now given by the attenuation matrix $\mathbf{H}^{(t)}$, where each matrix element $h_{j,n}^{(t)}$ represents the attenuation that terminal j experiences on sub-carrier n at time t.

$$\mathbf{H}^{(t)} = \left(h_{j,n}^{(t)} | \forall j, n\right) \tag{4.1}$$

As the dynamic scheme under consideration operates on an FDMA basis, different sub-carriers are assigned to different terminals. The specific assignment of sub-carrier n to terminal j at time t is a variable of the system. Denote each assignment by the binary variable $x_{j,n}^{(t)}$, where

$$x_{j,n}^{(t)} = \begin{cases} 1 & \text{if } n \text{ is assigned to } j \text{ at } t \\ 0 & \text{if } n \text{ is not assigned to } j \text{ at } t. \end{cases} \tag{4.2}$$

The set of all assignment variables $x_{j,n}^{(t)}$ forms the binary assignment matrix $\mathbf{X}^{(t)}$ – as the power vector $\vec{P}^{(t)}$ does for the power distribution among the sub-carriers. Depending on the power assignment $p_n^{(t)}$ for each sub-carrier n one of the M modulation/coding combinations is applied.

In such a system model, a straightforward optimization approach is to maximize the overall bit rate of the cell per down-link phase, as given in Equation (Multi-User Raw Rate Maximization).

$$\begin{aligned}
\max_{\vec{P}^{(t)},\mathbf{X}^{(t)}} \quad & \sum_{\forall j} \sum_{\forall n} F\left(\frac{p_n^{(t)} \cdot \left(h_{j,n}^{(t)}\right)^2}{\sigma^2}, p_{\text{err}}\right) \cdot x_{j,n}^{(t)} \\
\text{s.t.} \quad & \sum_{\forall j} x_{j,n}^{(t)} \leq 1 \qquad \forall n \\
& \sum_{\forall n} p_n^{(t)} \leq P_{\max}
\end{aligned} \tag{Multi-User Raw Rate Maximization}$$

Two general constraints exist for this problem. While one constraint limits the overall transmit power as in the case of problem (Bit Rate Maximization) for the adaptive loading, the other one is specific to the multi-user scenario. This is the requirement that each sub-carrier can be assigned to at most one terminal at a time. It has been shown that this requirement yields a significantly superior performance [38][2].

The optimization problem (Multi-User Raw Rate Maximization) is easy to solve: Assign each sub-carrier to the terminal with the lowest attenuation [38]. Then, a loading algorithm is applied in order to distribute the transmit power with respect to the objective function. Both steps are computationally cheap. However, this approach is not suitable in many cases. Unless all terminals are located quite close to the access point (for example, as in the case of a very small cell), some terminals will be much closer to the access point than others. Due to the path loss, this leads to much lower sub-carrier attenuations of the closer terminals than for the terminals far away. As a consequence, these "weak" terminals experience high delays for the reception of packets, if they receive anything at all. Thus, instead of considering the optimal solution to (Multi-User Raw Rate Maximization), two different problems have been considered in the literature, known as *margin adaptive* and *rate adaptive* approaches in dynamic OFDMA [39].

In case of the rate adaptive approach, for each down-link phase the throughput of each terminal is maximized (by maximizing a lower bound of all terminals' throughput) with respect to a transmit power budget. The problem formulation is given in Equation (Rate Adaptive).

$$\begin{aligned}
\max_{\vec{P}^{(t)},\mathbf{X}^{(t)}} \quad & \epsilon \\
\text{s.t.} \quad & \sum_{\forall j} x_{j,n}^{(t)} \leq 1 \quad \forall n \\
& \sum_{\forall n} p_n^{(t)} \leq P_{\max} \\
& \sum_{\forall n} F\left(\frac{p_n^{(t)} \cdot \left(h_{j,n}^{(t)}\right)^2}{\sigma^2}, p_{\text{err}}\right) \cdot x_{j,n}^{(t)} \geq \epsilon \quad \forall j
\end{aligned} \tag{Rate Adaptive}$$

For the margin adaptive problem each terminal j is considered to require a certain bit rate, which

[2] In this study the authors propose to assign a sub-carrier to multiple terminals. However, it turns out that exclusive assignments are superior.

translates into a certain amount of bits $r_j^{(t)}$ required per down-link phase. The objective is to minimize the overall transmit power while achieving the terminal's rate requirements.

$$
\begin{aligned}
\min_{\vec{P}^{(t)},\mathbf{X}^{(t)}} \quad & \sum_{\forall j} \sum_{\forall n} p_n^{(t)} \cdot x_{j,n}^{(t)} \\
\text{s.t.} \quad & \sum_{\forall j} x_{j,n}^{(t)} \leq 1 \qquad \forall n \\
& \sum_{\forall n} F\left(\frac{p_n^{(t)} \cdot \left(h_{j,n}^{(t)}\right)^2}{\sigma^2}, p_{\text{err}}\right) \cdot x_{j,n}^{(t)} \geq r_j^{(t)} \qquad \forall j
\end{aligned}
\tag{Margin Adaptive}
$$

Both approaches, the margin and rate adaptive ones, belong to the group of combinatorial optimization problems (i.e. integer programming problems), which are generally known to be hard to solve. Often, these specific problems have been claimed to be *NP*-hard, however, this has never been proven. In fact, both problems are *NP*-complete. The proof is provided in the appendix of this paper.

As a consequence, a significant computational overhead can be expected at the access point to solve them *optimally*. However, it has been shown that the performance gain due to dynamic OFDMA is quite large compared to OFDM systems which assign sub-carriers statically (either in TDMA or FDMA)[3]. Thus, so far the main focus has been on evaluating sub-optimal algorithms for dynamic OFDMA.

4.2 Generating Optimal and Suboptimal Solutions

In general, all proposals for the rate- or margin adaptive optimization problem belong to one of three different methods. The first one is to relax the integer constraint on the bit- or sub-carrier assignments [39, 40]. Thus, each sub-carrier can now carry a non-integer amount of bits or can be assigned to multiple different terminals during one down-link phase. By relaxing the integer constraint, the rate- and margin adaptive optimization problems become linear programming problems, which can be solved efficiently. However, after solving the relaxed problem, integer assignments have to be generated. Usually, this is done by reassigning the sub-carriers to the terminals with the largest non-integer fraction.

Following the second proposal, the optimization problem is split into two steps [41, 42]: First, each terminal is allocated a certain number of sub-carriers $m_j^{(t)}$ (referred to as sub-carrier allocation). Then, the specific sub-carriers are assigned to the terminals, i.e. the specific sub-carrier/terminal pairs are generated. Once the allocation of sub-carriers is given, the resulting optimization problem (Assignment Problem) can be solved efficiently. the sub-carrier allocations.

[3]Previously, the notion of static systems was defined as systems not adapting to channel variations. In the multi-user case we refer to a static system as one which does not adapt the sub-carrier assignments to the current channel states. Thus, fixed sub-carrier blocks are assigned to terminals. Still, an adaptive modulation scheme might be applied.

$$\begin{aligned}
\max_{\mathbf{X}^{(t)}} \quad & \sum_{\forall j} \sum_{\forall n} h_{j,n}^{(t)} \cdot x_{j,n}^{(t)} \\
\text{s.t.} \quad & \sum_{\forall j} x_{j,n}^{(t)} \leq 1 \qquad \forall n \\
& \sum_{\forall n} x_{j,n}^{(t)} \leq m_j^{(t)} \qquad \forall j
\end{aligned} \qquad \text{(Assignment Problem)}$$

This problem has a graph-theoretic counterpart: The bipartite weighted matching problem [43], which can be solved by the *Hungarian algorithm* [44] that has a complexity of $O\left(N^4\right)$ (where N equals the total number of sub-carriers in the system). However, more recent results show that the bipartite, weighted matching problem can be solved with a lower complexity of $O\left(N^3\right)$ [43], which is the fastest optimal algorithm known today.

As third approach it is suggested to solve the margin- or rate adaptive problem by heuristics [45, 46] mostly based on sorting procedures. Also, some contributions apply the technique of local search to the considered problem [47, 48]. Local search algorithms make use of an initial (hopefully "good") solution which is then iteratively improved by a local criteria [49]. In the following we present some approaches in detail, solving the problems of (Rate Adaptive) and (Margin Adaptive).

In 1999, Wong et al. [40] were the first to consider the optimization of a dynamic OFDMA system. They focused on the margin adaptive approach, as they were interested in reducing the intercell interference. The authors propose to apply integer relaxation. However, the resulting optimization problem is continuous but non-linear, such that the method of Lagrangian multipliers [18] is applied. The non-linearity stems from the assumed relationship of transmit power and data rate ($F\left(\mathrm{SNR}, p_{\mathrm{err}}\right)$). As with the solution of the Lagrangian equation system in the case of (Finite Tones Water-Pouring), some iterative computation is required to obtain a valid optimal solution. The resulting optimal assignment of sub-carriers to terminals and of power and bit rates to sub-carriers serves as lower bound. In a real transmission system no continuous relationship between transmit power and bit rate can be achieved. Also, non-integer assignments of sub-carriers to terminals might not be suitable for the considered system. Therefore, the authors propose to quantize the assignment results. Thus, a sub-carrier is always assigned to the terminal with the largest share. Afterwards, a power- and bit-loading scheme is applied. his initial work by Wong et al. serves as comparison basis for multiple later studies on the margin-adaptive problem. One such approach is presented by Kivanc et al. in [42]. It is based on the two-step approach: *Resource Allocation* and *Sub-Carrier Assignment*. Resource Allocation (determining the number of sub-carriers each terminal should receive) is done using the greedy BABS algorithm (cf. Algorithm 2). Once the resource allocation is determined for each terminal, the specific assignment of the sub-carriers is done by the Amplitude Craving Greedy (ACG) algorithm (cf. Algorithm 3). Simulations show that the power requirements of the combination BABS/ACG are only slightly higher than the power requirements of Wong's approach [40] while CPU run times are smaller by a factor of 100.

The first ones to consider efficient algorithms regarding the rate adaptive problem were H. Yin et al. in [41]. In order to solve it, the authors rely on the two-step approach. They propose to combine the allocation of power and sub-carrier amounts per terminal in the first step (cf. Algorithm

Algorithm: Resource Allocation Algorithm BABS

Result : Optimal Resource Allocation (number of sub-carriers) among users

Given the individual terminal rate requirements $r_j^{(t)}$*, the maximum rate per sub-carrier* R_{max}*, the attenuation matrix* $\mathbf{H}^{(t)}$ *and a rate-power function* $F^{-1}(\cdot)$ *(delivering the transmission power, according to the number of bits to send, the minimum BER requirements and available coding/modulation schemes), let terminal* j *be allocated* $m_j^{(t)}$ *sub-carriers at time* t *as follows:*

Calculate the initial allocation number per terminal j: $m_j^{(t)} = \lceil \frac{r_j^{(t)}}{R_{max}} \rceil \quad \forall j$

while $\sum_j m_j^{(t)} > N$ **do**

  Search for $j^* = arg \min_j m_j^{(t)}$

  Set $m_{j^*}^{(t)} = 0$

end

Calculate the average sub-carrier attenuation per terminal j: $h_{j,ave}^{(t)}$.

while $\sum_j m_j^{(t)} < N$ **do**

  Calculate the difference in transmission power needed when an additional sub-carrier is allocated to terminal j: $\Delta p_j^{(t)} = \frac{m_j^{(t)}+1}{h_{j,ave}^{(t)}} F^{-1}\left(\frac{r_j^{(t)}}{m_j^{(t)}+1}\right) - \frac{m_j^{(t)}}{h_{j,ave}^{(t)}} F^{-1}\left(\frac{r_j^{(t)}}{m_j^{(t)}}\right) \quad \forall j$

  Search for $j^* = arg \min_j \Delta p_j^{(t)}$

  Set $m_{j^*}^{(t)} = m_{j^*}^{(t)} + 1$

end

Algorithm 2: The BABS Algorithm.

4), where the allocation is based on the average channel to noise ratio of all sub-carriers for each terminal. The remaining problem to be solved in the second step is the assignment problem, given by problem (Assignment Problem). The authors suggest to use the Hungarian algorithm [44]. Once the assignment phase has been concluded, adaptive loading is applied to each terminal in order to efficiently distribute the terminal's allocated power among the assigned sub-carriers. hee et al. [45] come up with another approach to solve the (Rate Adaptive) problem. Apart from proposing integer relaxation, the authors present a quite simple heuristic which is based on a *constant* power assignment for all sub-carriers (cf. Algorithm 5). They claim that the constant power distribution does not reduce the systems performance too much due to the fact that most sub-carriers assigned will be in quite a good state. It is well known that at least for wireless point-to-point connections constant power distribution achieves almost the same performance as power loading (as discussed in Section 3).

Apart from these four contributions a lot more proposals have been published up to date. Further approaches regarding the margin-adaptive problem can be found in [48, 50–54]. Further results on the rate-adaptive approach can be found in [39, 46, 55].

Algorithm: Sub-carrier Assignment Algorithm ACG

Result : Near Optimal Distribution of Sub-Carriers among Users

Given the number of sub-carriers allocated $m_j^{(t)}$ *and the set of sub-carriers allocated* A_j *for each terminal* j *(where* $\#A_j$ *denotes the cardinality of the set* A_j*), determine the sub-carriers assignments:*

Initialize: $A_j = \{\} \quad \forall j$
for n=*1:*N **do**
  Search for $j^* = arg \min_j h_{j,n}^{(t)}$
  while $\#A_j = m_j^{(t)}$ **do**
    Gate the for this terminal: $h_{j,n}^{(t)} = 0$
    and search again: $j^* = arg \min_j h_{j,n}^{(t)}$
  end
  Set $A_{j^*} = A_{j^*} \cup \{n\}$
end

Algorithm 3: The ACG Algorithm.

Algorithm: Yin's combined Resource Allocation Algorithm

Result : Optimal Resource Allocation (number of sub-carriers and power) among terminals

Given the individual terminal rate requirements $r_j^{(t)}$*, the maximum transmittable power* $P_{\max}$*, the average channel-gain values* $g_j(t)$ *per terminal and a rate-power function* $F^{-1}(\cdot)$ *(delivering the transmission power, according to the number of bits to send, the minimum BER requirements and available coding/modulation schemes).*

Let each terminal j initially be allocated $m_j^{(t)} = 1$ sub-carrier.

Compute the overall number of allocated sub-carriers: $N_a = \sum_{j=1}^{J} m_j^{(t)} = J$.

Determine the power needed for each terminal to fulfill its rate requirement using only that sub-carrier: $P_j = m_j^{(t)} \cdot f^{-1}\left(\frac{r_j^{(t)}}{m_j^{(t)}}\right) / g_j(t)$.

Determine the total amount of power needed to fulfill all data-rate requirements: $P_a = \sum_{j=1}^{J} P_j$.

while $P_a > \frac{N_a}{N} P_{\max}$ **do**

- Compute $\Delta P_j = m_j^{(t)} \cdot f^{-1}\left(\frac{r_j^{(t)}}{m_j^{(t)}}\right) / g_j(t) - \left(m_j^{(t)} + 1\right) \cdot f^{-1}\left(\frac{r_j^{(t)}}{\left(m_j^{(t)}+1\right)}\right) / g_j(t)$.
- Select $j^* = \arg\min_{j \in J} \Delta P_j$.
- Update $m_{j^*}^{(t)} = m_{j^*}^{(t)} + 1$ and $P_{j^*} = P_{j^*} - \Delta P_{j^*}$.
- Update N_a and P_a.

end

Allocate the remaining amount of sub-carriers $N - N_a$ to the terminal j^* with the highest average channel gain value g_{j^*}

Algorithm 4: Combined power and sub-carrier allocation algorithm.

Algorithm: Rhee's combined Allocation and Assignment Algorithm

Result : Optimal terminal/sub-carrier assignment sets.

Given a scenario of J terminals sharing N sub-carriers in one cell, the individual sub-channel-gain values $g_{j,n}^{(t)}$ per terminal and the Shannon Capacity function $F(\cdot, p_{\mathrm{err}})$ (delivering the capacity of a channel, depending on the channel gain):

Set the momentary data-rate per terminal $r_j^{(t)} = 0$ for all $j \in J$.
Let A be the set of available sub-carriers $A = \{1, 2, \ldots, N\}$.

for $j = 1$ *to* J **do**
    Find n^* such that $|g_{j,n^*}^{(t)}| \geq |g_{j,n}^{(t)}|$ for all $n \in A$ with $n \neq n^*$.
    Update $r_j^{(t)} = F\left(g_{j,n^*}^{(t)}, p_{\mathrm{err}}\right)$ and $A = A \backslash \{n^*\}$.
end
while $A \neq \emptyset$ **do**
    Find j^* such that $r_{j^*}^{(t)} \leq r_j^{(t)}$ for all j with $j \neq j^*$ and $0 \leq j \leq J$.
    For j^* find n^* such that $|g_{j^*,n^*}^{(t)}| \geq |g_{j^*,n}^{(t)}|$ for all $n \in A$ with $n \neq n^*$.
    Update $r_{j^*}^{(t)} = r_{j^*}^{(t)} + F\left(g_{j^*,n^*}^{(t)}, p_{\mathrm{err}}\right)$ and $A = A \backslash \{n^*\}$.
end

Algorithm 5: Combined allocation and assignment algorithm.

4.3 System Aspects of Dyn. Schemes for Point-to-Multi-Point Conn.

4.3.1 Performance Results

Again, the most important question in the point-to-multi-point scenario relates to the performance gain that can be achieved with a dynamic OFDMA approach. However, due to the fact that the margin- and rate-adaptive problems cannot be solved optimally in real time, the performance issue consists of two questions: What is the potential gain that the optimal solution provides and how much of this potential gain can be achieved by sub-optimal algorithms. In the following, we focus on the rate-adaptive approach in order to discuss these questions. Note that for the margin-adaptive approach quite similar statements can be made.

In [56] the authors investigated the potential gain for several optimal variants. In general sub-carriers as well as power and bits can be assigned dynamically to different terminals according to the chosen approach (margin- or rate adaptive). However, frequently it has been suggested to fix the transmit power per sub-carrier and only assign sub-carriers and bits dynamically. As shown in the complexity analysis in the appendix, this does not change the theoretical complexity of the rate adaptive approach. However, in practice this decreases the run-times of algorithms significantly, as the amount of feasible solutions is much lower. The question is how much *additional* performance is provided by power loading in the multi-user case. Accordingly, the question arises how much performance is provided by dynamically assigning only power and bits, while the sub-carriers are distributed statically.

In Figure 4.2 average throughput and delay results are given for a certain cell parameterization while increasing the cell's radius[4]. Potentially, the dynamic schemes outperform static schemes quite a lot, in terms of the average throughput the gain is up to $100\ \%$, even larger for the maximum delay in order to transmit an IP packet of size 1500 Bytes. It is important to note that the power adaption does yield a significant performance increase, which is much larger in the case of dynamic sub-carrier assignments than in the case of static sub-carrier assignments, especially regarding the average throughput.

How much of this optimal performance can be reached by suboptimal algorithms? This question is to some extent open. For example, following the two-step approach of Yin et al. [41] the optimum can be reached by about 90%. Rhee et al. [45] find for a fairly simple heuristic a much smaller performance difference of about 2%. Kim et al. [39] suggest that the relaxation strategy performs equally well when compared to the potential gain of the optimal solution. However, there are some inconsistencies with the results presented in all these studies. In addition, it is not clear how the performance behaves in different scenarios, i.e. for a varying transmit power, for a varying number of terminals, for a varying number of sub-carriers, etc.

In Figure 4.3 the average throughput of the two-step approach (following Yin et. al [41]), of the quantized integer relaxation (following Kim et al. [39]), and of the heuristic according to Rhee et al. [45] is compared to the optimal rate adaptive throughput with a static power distribution. The average throughput of the optimal performance is met quite well by all three suboptimal schemes. However, in terms of the delay, the quantized relaxation performs best, achieving almost the optimal performance. If also dynamic power distribution is enabled (Figure 4.4) in the system, all three suboptimal schemes are significantly outperformed by the optimal solution in terms of throughput as

[4] As the radius increases, the path loss spread between terminals at different positions increases. This makes the optimal solution of the rate adaptive approach more difficult.

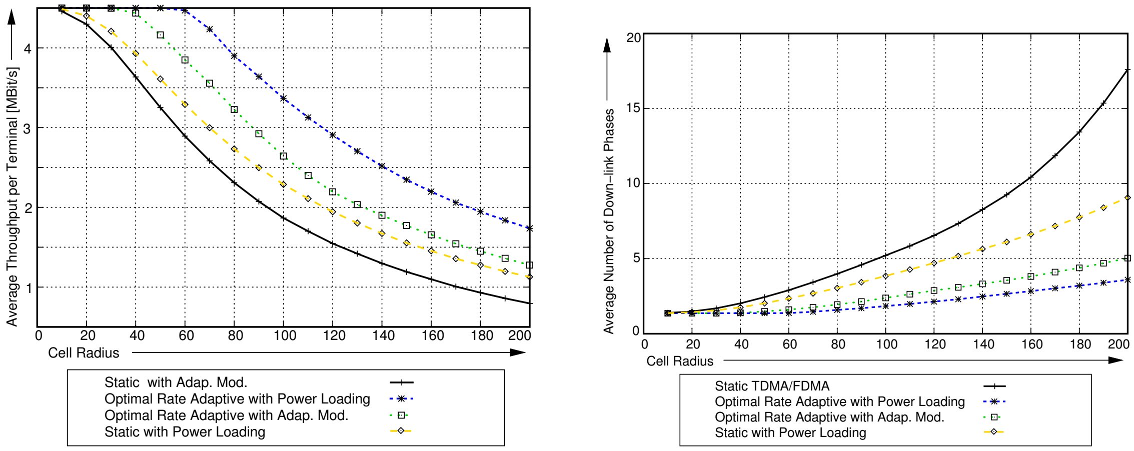
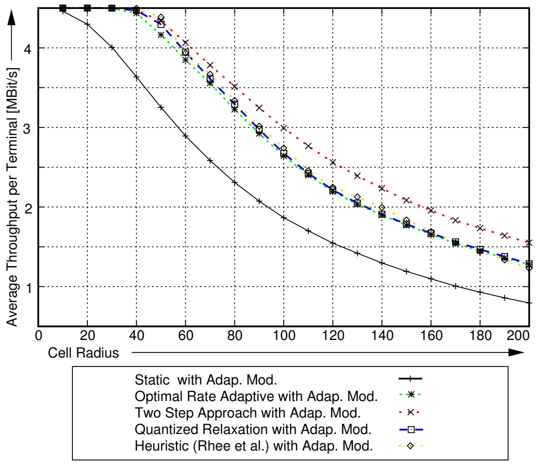


Figure 4.2: Average throughput and packet delay for a static OFDMA scheme, for static sub-carrier assignment with power loading, for dynamic sub-carrier assignment with static power distribution and for a fully dynamic scheme (always for the optimal solution of the rate adaptive approach). 8 terminals are located in the cell assuming a system bandwidth of 16.25 MHz divided into 48 sub-carriers, 4 different modulation types are available (BPSK, QPSK, 16-, and 64-QAM, target symbol error rate is 10^{-2}, the transmit power is set to 10 mW, one up-link and down-link phase has a duration of 1 ms applying a TDD mode.

well as in terms of the maximum delay.

Apart from considering the optimal performance of a dynamic OFDMA system, other studies consider the implementation aspects of such dynamic OFDMA systems. In [57] Rohling et al. consider different multiple access schemes for broadband OFDM systems. They find that dynamic OFDMA clearly outperforms TDMA and CDMA variants. However, FDMA also requires the largest overhead due to signaling, as is discussed below.

In a different paper [58] Wang et al. study various effects in combination with a certain system concept of a multi-user OFDM system in FDD. The authors propose a system where the time and frequency radio resources are divided into bins (chunks) of certain size (for example, 120 symbols obtained from combining 10 sub-carriers of bandwidth 10 kHz each and 12 symbols of length 111 μs each). Given perfect channel knowledge at the access point (of all terminal bins during the next three bins), the access point assigns bins to the terminal with the best channel attenuation. Then adaptive modulation is applied to the specific bin. The authors show in the simulation part how the system benefits from multi-user diversity and an increasing transmit power. Analytically, they find a method to obtain the spectral efficiency of the system and proof their results by means of simulation. Also, they present results on a varying size of the bin (indicating that there are significant consequences for the system's performance if the bin size is chosen too large or too small due to the caused overhead) and investigate the performance of the system at different average receiver power levels. No delay or rate requirements for each terminal are assumed.



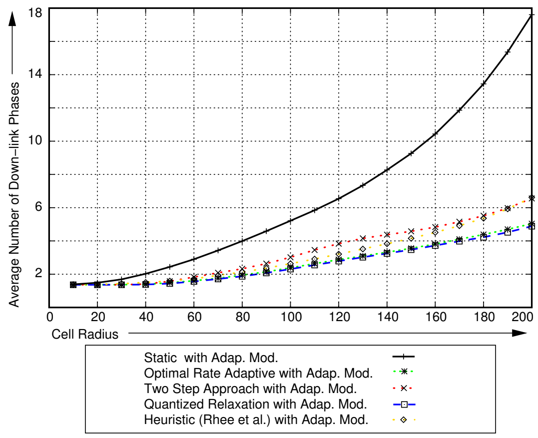


Figure 4.3: Average throughput and delay results for three different suboptimal approaches without power loading compared to a static and optimal scheme while varying the cell radius. 8 terminals are located in the cell assuming a system bandwidth of 16.25 MHz divided into 48 sub-carriers, 4 different modulation types are available (BPSK, QPSK, 16-, and 64-QAM, target symbol error rate is 10^{-2}, the transmit power is set to 10 mW, one up-link and down-link phase has a duration of 1 ms applying a TDD mode.

4.3.2 Overhead Issues

As dynamic OFDMA schemes provide a significant performance gain, they are also subject to a certain required overhead due to signaling. In the multi-user case, the signaling information contains the full assignment set (which terminal is assigned which sub-carrier with which modulation type). This can be quite costly, however it has been shown that it does not consume the complete performance gain achieved by the dynamic assignments. Instead, the signaling overhead is found to reduce the system performance by about 10% at worst in certain scenarios [59]. Interestingly, the signaling overhead varies with certain system parameters such as the number of terminals or the number of sub-carriers used per cell. This leads to a qualitatively different behavior of a dynamic OFDMA system compared to studies where the signaling overhead has not been considered.

Regarding the signaling overhead there are also attempts to reduce it by different means. One such opportunity is to build blocks of sub-carriers which are assigned to different terminals instead of assigning individual sub-carriers. This option has been investigated in [48, 58, 60], although the authors do not study the overhead decrease due to the block building of sub-carriers. However, these studies show that block building is quite limited as soon as the sub-carrier blocks are larger than the coherence bandwidth. In these cases the performance loss is already quite large, while the overhead decrease is still low (decreasing the number of sub-carriers by a factor of two reduces the signaling overhead *per single assignment* only by one bit). Alternatively, the signaling overhead can be incorporated into the optimization problem. This has been proposed in [61]. Here, quite similar assignments from one down-link phase to the next one can be generated due to the correlation in time. Thus, the overall overhead is decreased. However, this can lead to higher computational

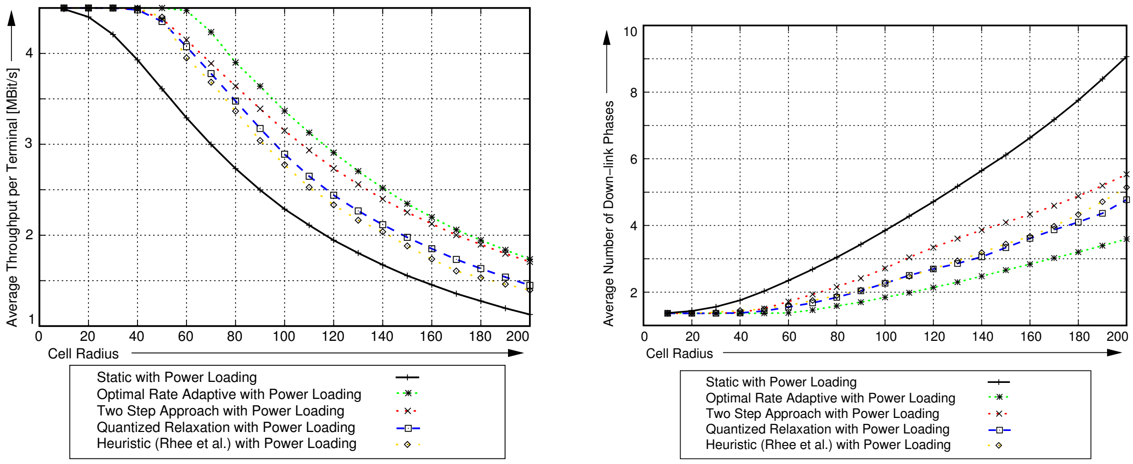


Figure 4.4: Average throughput and delay results for three different suboptimal approaches with power loading compared to a static and optimal scheme while varying the cell radius. 8 terminals are located in the cell assuming a system bandwidth of $16.25\ \mathrm{MHz}$ divided into 48 sub-carriers, 4 different modulation types are available (BPSK, QPSK, 16-, and 64-QAM, target symbol error rate is 10^{-2}, the transmit power is set to $10\ \mathrm{mW}$, one up-link and down-link phase has a duration of $1\ \mathrm{ms}$ applying a TDD mode.

resource requirements at the access point in order to generate these distinguished assignments.

Apart from signaling, the access point requires some knowledge regarding the sub-carrier attenuations. As for point-to-point connections, this is rather easy to obtain in TDD systems, as the wireless channels can be assumed to be reciprocal. However, in FDD systems quality reports of the sub-carriers have to be transmitted in the up-link, causing a performance loss in this transmit direction. This issue has not been studied so far.

Chapter 5

Conclusions and Future Work

The performance of wireless OFDM systems can be significantly increased if the transmitter and receiver pair adapt constantly to the current channel conditions. For point-to-point connections the transmitter generates a power and/or modulation (possibly including also coding) assignment per sub-carrier. Sub-carriers with a relatively low attenuation convey more information, sub-carriers with a relatively high attenuation contribute less to the transmission. It has been shown that such schemes lead to a much lower bit error rate, a much lower transmit power and/or to a much higher throughput. This comes at the cost of more computational resources required at the transmitter and the exchange of control information, namely for signaling (conveyed from the transmitter to the receiver) and for channel knowledge (conveyed from the receiver to the transmitter). Unless the channel is extremely time-selective, the performance gain justifies the additional cost.

In the case of point-to-multi-point connections, the cost is higher. In addition to the power and modulation assignment per sub-carrier, the available sub-carriers have to be assigned to multiple terminals. The resulting optimization problems (namely the rate and margin adaptive approaches) are *NP*-complete problems, as shown in this paper. Despite the relatively high cost, the potential performance increase achieved by dynamic OFDMA schemes is quite high (about 100% and more). Thus, many sub-optimal schemes have been studied recently, for example the two-step approach or linear relaxation. If the transmit power per sub-carrier is fixed, suboptimal schemes perform quite well. However, in the case of a variable transmit power per sub-carrier, all so far discussed sub-optimal schemes are significantly outperformed by the optimum. Thus, better suboptimal schemes are of interest for such systems, especially as dynamic power adaption provides quite a performance gain. There is also much more overhead due to control information in the case of point-to-multi-point connections compared to point-to-point connections. However, schemes such as compression or optimization can be applied to the signaling information, which reduce the overhead quite well. Still, the performance loss due to signaling is about 5% or above. Regarding the channel knowledge, the overhead problem arises mainly in the case of the FDD mode. Little is known about the issue of channel knowledge. Compression schemes might work quite well to reduce the overhead due to channel knowledge in FDD systems, however, this has never been studied so far.

Some more issues remain as further work as well. First of all, it is not known how fast the suggested algorithms perform their assignments on rather low-cost computing equipment. Any delay due to computation degrades the systems performance. While the attenuation of wireless channels varies on the time scale of milliseconds, computation time spans down to a few microseconds are of interest which still provide a high system performance. This is a very demanding challenge.

Furthermore, most optimization models presented so far do not consider the fact that packets have to be transmitted instead of arbitrary amounts of bits. In addition to this, future systems are also characterized by packets belonging to different quality of service classes, multiple antenna transmission techniques and sophisticated error correction strategies, i.e. hybrid ARQ. Despite the fact that dynamic OFDMA systems are considered for future cellular communication systems, all these issues related to more “realistic” models have not been considered so far. Thus, entirely new optimization models might be needed, requiring different solution strategies.

Appendix

We consider (Rate Adaptive Maximization – fixed Power), which is a special case of the original rate-adaptive optimization problem: Instead of a dynamic power assignment the transmit power per sub-carrier is fixed. The fixed transmit power yields together with the attenuation per sub-carrier and the noise an SNR value. Given a target bit error probability, for each SNR value a certain modulation type is obtained. Thus, the attenuation matrix $\mathbf{H}^{(t)}$ can be transformed into a bit matrix $\mathbf{B}^{(t)}$. This bit matrix denotes the number of bits which can be transmitted from the access point to each terminal on each sub-carrier during the next down-link phase (all elements of $\mathbf{B}^{(t)}$ are integer values).

$$\begin{aligned} \max_{\mathbf{X}^{(t)}} \quad & \epsilon \\ \text{s.t.} \quad & \sum_{\forall j} x_{j,n}^{(t)} \leq 1 \quad \forall n \\ & \sum_{\forall n} b_{j,n}^{(t)} \cdot x_{j,n}^{(t)} \geq \epsilon \quad \forall j \end{aligned} \qquad \text{(Rate Adaptive Maximization – fixed Power)}$$

This optimization problem is *NP*-complete. For a detailed introduction to *NP*-completeness proofs in the context of optimization problems, refer to [49, 62]. First, the *recognition version* of this rate-adaptive optimization problem (Rate Adaptive Maximization – fixed Power) is required:

> **HEAVY DISJOINT SUBSETS :**
> Given N sets of J integers $B_n = \{b_{1,n}, \dots b_{J,n}\}$ and an integer L, are there J disjoint subsets $A_j \subseteq \{1, \dots N\}$ such that $\forall j : \sum_{\forall n \in A_j} b_{j,n} \geq L$?

This is the recognition problem of (Rate Adaptive Maximization – fixed Power), as the N different sub-carriers of the system are represented by the integer sets B_n, each one having J states for the terminals in the system. The assignments per terminal are recorded in the integer sets A_j, which have to be disjoint such that each sub-carrier is assigned only once.

In order to prove that this recognition problem is *NP*-complete, we first show that all problems in *NP* polynomially reduce to HEAVY DISJOINT SUBSETS. In a second step we show that HEAVY DISJOINT SUBSETS belongs to the class of *NP*. First consider the reduction. For the reduction the *NP*-complete problem PARTITION [62] is chosen:

> **PARTITION :**
> Given J integers $c_1, \dots, c_J$, is there a subset $S \subseteq \{1, \dots, J\}$ such that $\sum_{\forall j \in S} c_j = \sum_{\forall j \notin S} c_j$?

Suppose now we have an algorithm $\mathfrak{A}$ solving the recognition problem HEAVY DISJOINT SUBSETS. We are given a problem instance of PARTITION (the set of integers $\{c_1, \dots, c_J\}$). This instance of PARTITION can be solved by calling $\mathfrak{A}$ with a specific parameterization. The reduction works as the following: Denote by $C = \{c_1, \dots, c_J\}$ the set of given integers. As input $\mathfrak{A}$ receives $N = 2$ sets of J identical integers $B_1 = C$ and $B_2 = C$. Prior to invoking $\mathfrak{A}$, we check the sum $l = \sum_{\forall j \in \{1,\dots,J\}} c_j$ if it is even or odd. In the case that l is odd, we stop immediately since this instance of PARTITION can not be solved. Otherwise we set $2 \cdot L = l$, where L is the lower limit used in HEAVY DISJOINT SUBSETS.

Algorithm $\mathfrak{A}$ will now try to find two disjoint subsets such that the "weight" of each subset is greater or equal half of the sum l of the PARTITION integers. Thus, using $\mathfrak{A}$ and this spe-

cific way of parameterizing it with an instance of PARTITION, we solve the *NP*-complete problem PARTITION. Generating the special parameterization for algorithm $\mathfrak{A}$, given the instance of PARTITION, can obviously be done in polynomial time. Therefore, we have found a polynomial time reduction of PARTITION to HEAVY DISJOINT SUBSETS, the recognition version of problem (Rate Adaptive Maximization – fixed Power). Hence, HEAVY DISJOINT SUBSETS is *NP*-hard and therefore the optimization problem
(Rate Adaptive Maximization – fixed Power) is *NP*-hard, too.

It remains to show that HEAVY DISJOINT SUBSETS belongs to the class of *NP*. Consider a potential solution of an instance of this problem. It can be represented by denoting N pairs (n, j), referring to the integer $b_{j,n}$ which was chosen. This certificate is polynomially bounded in space as it grows linearly with N. This certificate can also be validated quite fast. Skipping through the set of pairs, for each j the integer $b_{j,n}$ is summed separately. Next, each of these summations are checked if they are greater or equal L. If so, then it remains to validate that of each set B_n only one member has been chosen. Therefore, while skipping through the pairs also each n is added to a sorted list. If for some n this list already contains an entry, the potential solution is discarded. All these operations are polynomially bounded if varying the parameters N or J. Therefore, it can be concluded that HEAVY DISJOINT SUBSETS is *NP*-complete.

As problem (Rate Adaptive Maximization – fixed Power) is just a special case of the general rate-adaptive problem (Rate Adaptive), it is also *NP*-complete. In addition, the recognition version of problem (Rate Adaptive) is similar to the recognition version of problem (Margin Adaptive). As a consequence, the margin adaptive optimization problem is *NP*-complete as well.

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