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Channel State Dependent Scheduling Policies for an OFDM Physical Layer using a M–ary State Model

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Abstract

In this report we investigate simple scheduling mechanisms to be used in wireless networks utilizing an **O**rthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) physical layer. The assumed scenario for the wireless network corresponds very much with the one used in [7]. In contrast to [7], the focus of this report is to use a more realistic model of the behavior of the sub–carrier states. Therefore a model with M different possible sub–carrier states for each wireless terminal is introduced. Each state is related to a certain modulation, such that transmission data rates for each modulation type are different. Again we compare dynamic scheduling policies with a static scheme equal to pure **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*) . Beside throughput, which represents the main metric of performance, constraints are also on signaling, fairness and complexity. Simulations and theoretical analysis show a significant performance increase for the dynamic scheduling policies compared to the static one. As signaling increases for the dynamic scheduling policies, the trade off between signaling overhead and throughput increase has to be taken into consideration.

Contents

Chapter 1

Introduction

The demand for wireless communication systems has increased exponentially in the last few years [6]. While telephony was the major application for wireless networks of the first and second generation, third generation mobile networks will provide in addition data services with limited throughput. **I**nter**s**ymbol **I**nterference (*ISI*) due to multi–path propagation is one of the major issues limiting the maximal throughput. In order to avoid this shortcoming, multi–carrier transmission schemes have been proposed within the last couple years as next generation physical layer transmission scheme. In particular **O**rthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) has received special interest. By splitting a frequency band into a lot of sub–carriers and transmitting data through all these sub–carriers with data rates being much smaller compared to the overall data rate, the single symbol duration becomes large such that ISI is no longer a critical issue.

One future application of OFDM will be multi–user scenarios, for example cellular systems. For such scenarios a multiplexing scheme for the down–link and a **M**edium **A**ccess **C**ontrol (*MAC*) scheme for the up–link is necessary. In general one of the widely known schemes may be applied, which are **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*) , **T**ime **D**ivision **M**ultiple **A**ccess (*TDMA*) and **C**ode **D**ivision **M**ultiple **A**ccess (*CDMA*) . While scheduling schemes based on an OFDM–TDMA system already have been investigated [2], scheduling schemes based on an OFDM–FDMA system have not been described before. In an OFDM–FDMA system, the complete set of sub–carriers is divided into several subsets of sub–carriers. Afterwards these subsets of sub–carriers are assigned to wireless terminals.

While in [7] the main focus was on finding dynamic scheduling policies being able to outperform a static scheduling policy equal to pure **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*) , the purpose of the investigations in this report is different. Still, pure FDMA serves as comparison scheme, but here the impact of more realistic statistical models for the sub–carrier state behavior is of primary interest. Therefore this report may be seen as an extension to [7]. In particular, instead of using a sub–carrier state model with two possible states for each sub–carrier towards some wireless terminal, a model is introduced with M different possible states. The states depend on the actual **S**ignal–to–**N**oise **R**atio (*SNR*) of the sub–carrier towards the wireless terminals. In addition, related to each of the M possible states M different modulation schemes are introduced, which may convey data at different rates. This is in contrast to [7], where only one modulation type was available. Therefore, this new model introduces the possibility of multiple states and adaptive modulation and represents so a more realistic sub–carrier model than it was the case with the binary sub–carrier state model. The question of main interest is, if dynamic scheduling policies may be found for this model performing as good as the ones found for the binary sub–carrier state model.

The remaining technical report is structured as the following. In 2 we introduce the new sub–carrier state model and the assumptions related to it. Afterwards we introduce in 3 the scheduling policies for a specific form of the new sub–carrier model and adjust some of their properties to the new model. There we also give some analytical expressions for the throughput. Then we compare the policies for an example setting and discuss afterwards some advanced versions of the dynamic scheduling policy. Also we give some theoretical bounds on the throughput. In 4 we do exactly the same for a second variant of the new sub–carrier model. At the end, we give some conclusions in 5.

For an introduction to OFDM as modulation scheme refer to [7]. There we also give an introduction to scheduling in wireless systems and to channel–state dependent scheduling policies.

Chapter 2

Scenario Description and Assumptions

Let us assume a wireless cellular system. The physical layer is using **O**rthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) as transmission scheme. For each cell an access point organizes all downstream and upstream data transfers regarding the wireless terminals located in that cell. In general we assume a total amount of J wireless terminals to be located within the cell. We focus on down–link transmissions to achieve upper–bound results. In particular we assume that each wireless terminal has exactly one data transfer session active. All J data streams are equal in their quality of service requirements. The bit rate is considered to be constant. For the further assumptions of the scenario as well as for the parameters, the performance metrics and the fairness and complexity constraints, refer to [7].

In contrast to [7] the statistical model assumed for the sub–carrier state behavior is now different. In [7] we considered a binary sub–carrier state model which possessed two states for each sub–carrier towards each wireless terminal. However, in general more possible states per sub–carrier towards a single wireless terminal exist. For example in **HI**gh **P**erformance **R**adio **L**ocal **A**rea **N**etwork (*HIPERLAN*) a sub–carrier may be in one of seven different states towards a wireless terminal [8]. These seven different states are characterized by seven different bit rates, which are a result of using one of four different modulation types and puncturing the convolutional coding with two different rates. Therefore we introduce here a sub–carrier state model offering M states with exactly one modulation scheme for each state possible. This is the general model foundation which will be assumed later on in the report in order to discuss the performance for different scheduling policies.

I detail we focus here on two different models, each including the M possible states per sub–carrier. These two models differ only in the distribution shape of their SNRs. As first model we will deal with the case that the SNRs of the sub–carriers are distributed uniformly. We will bound this continuous distribution by an upper and lower SNR value. Let us denote these values by SNR_{max} and SNR_{min}. As stated above, we will deal with M modulation types. Since we assume in general that the symbol time remains constant [7], different modulation types vary only in the amount of bits that are represented by one symbol. The usage of a certain modulation type is dependent on a minimal required sub–carrier SNR. In case that multiple modulation types are possible (which is in fact almost always the case, except for a SNR value below the threshold for the modulation type with the second lowest represented amount of information per symbol), always the one with the highest bit rate is chosen. Therefore every modulation type has a SNR zone for which it will be applied to a sub–carrier if its estimated SNR lies within this zone.

For the width of these zones and the related transmission rates we assume the following. For the M zones let the width of the SNR zones be identical, therefore we obtain for the SNR width a value of $\frac{SNR_{max}-SNR_{min}}{M}$ for each zone. For the best SNR zone (zone M) the used modulation will have a symbol information length of b bits. For the worst SNR zone (zone 1) the used modulation will have only a symbol length of $\frac{b}{M}$ bits. Therefore the amount of information conveyable on a sub–carrier being in the best possible SNR zone will be M–times bigger than the amount of information being conveyable on a sub–carrier in the worst SNR zone. For the SNR zones in between the used modulation symbol length will be linearly dependent on the SNR zone order number. For example the second worst SNR zone (zone 2) is related to a modulation with a symbol length of $\frac{2 \cdot b}{M}$ bits and so on. In general the z worst zone will be related to a modulation scheme with an information amount of $\frac{z}{M} \cdot b$ bits per symbol. Figure 2.1) shows the relationship between uniform SNR distribution, SNR zone numbering and used modulation symbol length for the case of $M = 8$.

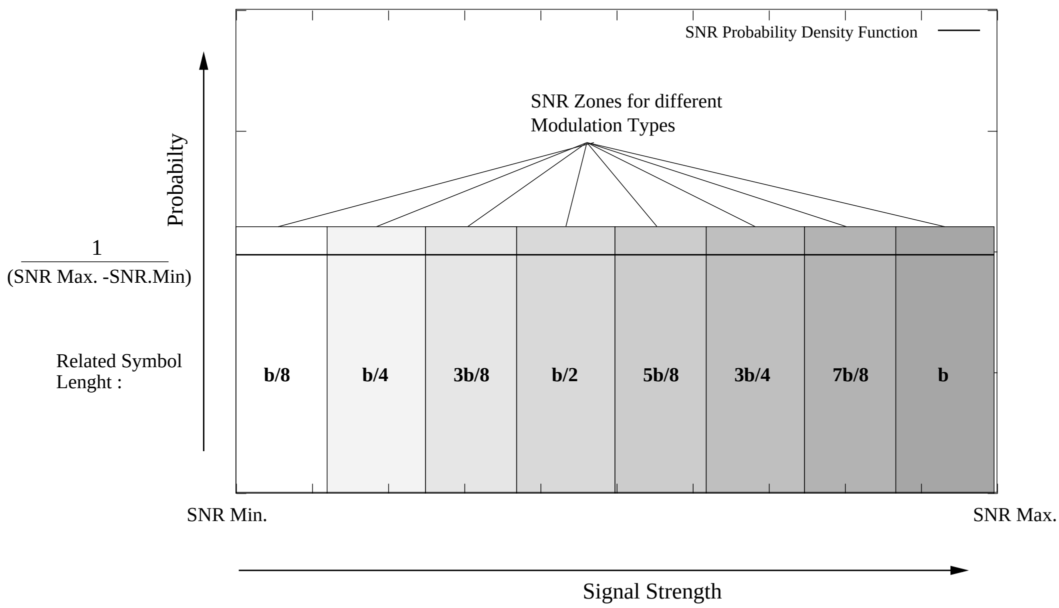

Figure 2.1: Relation between SNR Zones and Modulation Symbol Lengths for the M–ary Sub–Carrier State Model with an uniform SNR Distribution, here with with M = 8

If we denote by p_i the probability that a sub–carrier SNR towards a wireless terminal lies within zone i, then p_i will be equal for all M zones and will take the value $p_i = \frac{1}{M}$. In other words, all M transmission states for a sub–carrier have the same probability. We will refer to this sub–carrier state model as the M–ary sub–carrier state model with an uniform SNR distribution. The discussion of scheduling policy performances for this model will be presented in Section 3.

As second statistical model we will consider a sub–carrier state model where the SNR distribution is Gaussian rather than uniform (refer to Figure 2.2). Beside this everything is the same as in the M–ary sub–carrier state model with an uniform SNR distribution. Again M different modulation types are available. These M modulation types are bound to M sub–carrier SNR zones. The modulation type related to the worst SNR zone provides a symbol information length of $\frac{b}{M}$ bits per symbol. The modulation type related to the best SNR zone provides a symbol information length of b bits per symbol. The Gaussian distribution has a mean value of SNR_{Mean} and a variance of SNR_{Var}. For the SNR zones we will assume that they are still equally spaced where $\frac{M}{2}$ zones lie to the right of SNR_{Mean} and $\frac{M}{2}$ zones lie to the left of SNR_{Mean}. Each zone i has in general a lower SNR limit i_{low} and an upper SNR limit i_{up}. For the worst zone (lying at most left) the lower limit is $-\infty$ and for the best zone (lying at most right) the upper limit is ∞. With these definitions we can calculate the probability that a certain SNR value will lie in zone i. We obtain for p_i:

$$p_i = \int_{i_{low}}^{i_{up}} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x-SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx \tag{2.1}$$

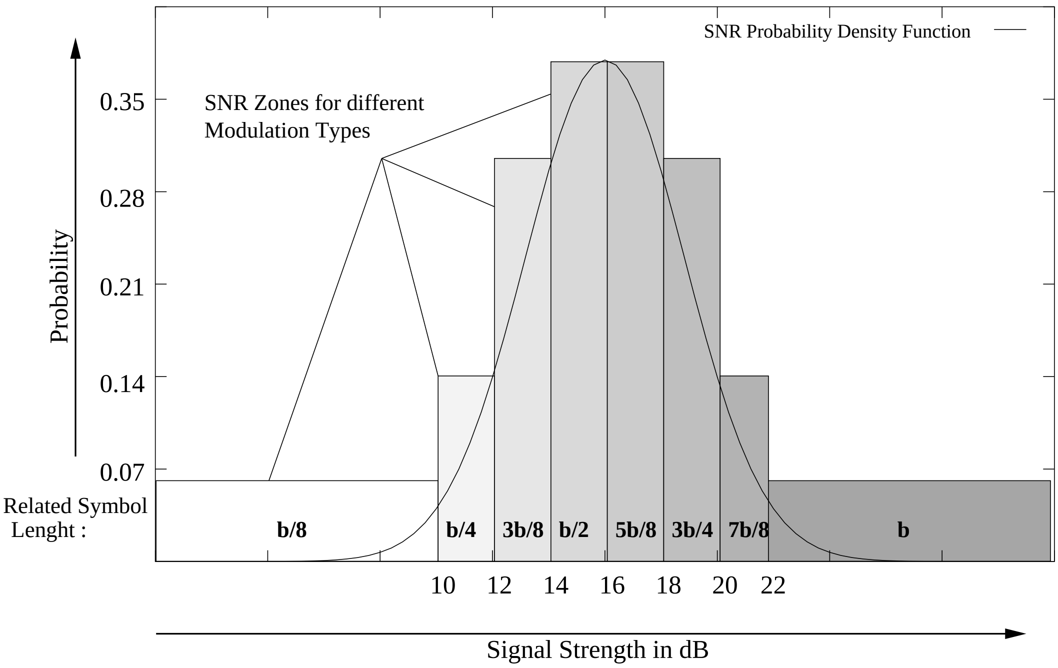

Figure 2.2: Relation between SNR Zones and Modulation Symbol Lengths for the M–ary Sub–Carrier State Model with a Gaussian SNR Distribution, here with with $M = 8$, $SNR_{Mean} = 16$dB and $SNR_{Var} = 4$dB

We will refer to this sub–carrier state model as the M–ary sub–carrier state model with a Gaussian SNR distribution. The discussion of scheduling policy performance for this model will be presented in Section 4.

The reason for assuming two different SNR distributions within the same statistical framework is rather simple. In [7] the statistical model was dependent on a certain probability that a sub–carrier is in a good state towards some wireless terminal. This variable was denoted by p_g. Depending on different values for p_g, a different behavior of the scheduling policies was observed. This could be true for the M–ary sub–carrier state model as well. Therefore in order to find out more about the behavior of the scheduling policies introduced afterwards, different SNR distributions were assumed. In addition to this, different environments may cause a different statistical behavior of a SNR distribution. However, a scheduling policy might not be aware of the specific SNR distribution, but should still work efficient. Thus, the throughput behavior for different SNR distributions is important in order to observe, if the scheduling policy is flexible.

Chapter 3

Scheduling Policies for the M–ary Sub–Carrier State Model with an uniform SNR Distribution

In this section we discuss all results related to the M–ary sub–carrier state model with an uniform SNR distribution. At first we present analytical results on a static assignment scheme. Then we show results for a simple dynamic scheduling policy. This policy can be advanced in order to increase the throughput. This is done in the successing section. At last, we also discuss theoretical upper bounds for this model and compare the presented scheduling policies to the upper bounds.

3.1 Static Sub–Carrier Assignments – SSA

As with the binary sub–carrier state model in [7], the mean throughput per wireless terminal in case that each sub–carrier is always in the best possible SNR state is $\frac{b}{T_s}$ bits. Now let us investigate the case of a sub–carrier behavior according to the M–ary sub–carrier state model described above. As comparison for the dynamic scheduling policies we will consider again the static sub–carrier assignment – SSA. For a detailed introduction of the static sub–carrier assignment refer to [7]. In order to obtain the mean throughput per wireless terminal, we will derive the mean throughput value per sub–carrier. Since we assume an uniform distribution of the SNR values, we obtain for each SNR zone a probability of $p_i = \frac{1}{M}$ that the SNR of a sub–carrier will lie in one of these zones. Therefore we obtain a mean throughput per sub–carrier of:

$$D_{\text{sub}} = \sum_{i=1}^{M} \left(p_i \cdot \frac{i}{M} \right) \cdot \frac{b}{T_s} = \sum_{i=1}^{M} \left(\frac{1}{M} \cdot \frac{i}{M} \right) \cdot \frac{b}{T_s} \tag{3.1}$$

Since a static assignment scheme consists only of a collection of N sub–carriers, the general throughput will be the mean value multiplied by the number of sub–carriers in the set N. We obtain therefore:

$$D_{\text{M–ary uniform SSA}} = \sum_{i=1}^{M} \left(\frac{1}{M} \cdot \frac{i}{M} \right) \cdot \frac{N \cdot b}{T_s} \tag{3.2}$$

3.2 Simple Rotating Sub–Carrier Space Algorithm – Simple RSSA

Now we will apply the rotating sub–carrier space algorithm to the M–ary sub–carrier state model with an uniform SNR distribution. For a detailed introduction of the rotating sub–carrier space algorithm refer to [7]. For illustration purposes we will set $J = 3$, therefore derive the mean throughput for 3 wireless terminals in the cell. The algorithm at the access point will now choose the N best available sub–carriers towards the actual priority class wireless terminal. This is in contrast to the rotating sub–carrier space algorithm for the binary sub–carrier state model, where sub–carriers could only be in either a good state or in a bad state and therefore the RSSA always only picked N sub–carriers being in a good state (if possible). In order to obtain the mean throughput for this scheduling policy, we will again have to derive the mean throughput for each priority class [7]. The mean throughput for each wireless terminal will be (for three wireless terminals) :

$$D_{\text{M–ary uniform SRSSA}} = \frac{D_{first} + D_{second} + D_{third}}{3} \tag{3.3}$$

In order to obtain the mean throughput for each priority class, we desire the mean throughput of every sub–carrier this priority class gets assigned. Recall that the wireless terminal in the first priority class obtains the N best sub–carriers out of $3 \cdot N$ sub–carriers in total. The mean throughput of each sub–carrier depends on the mean symbol information length the sub–carrier may convey. As we have seen in Section 2, the symbol information length depends on the SNR zone, the sub–carrier is actually in. Through the probability p_i where i denotes some SNR zone i, the probability of the related symbol information length is given. As shown, does the SNR zone i influence the symbol information length by adding a ratio to the maximal achievable symbol information length of b bits. SNR zone i will have a symbol length of $\frac{i}{M} \cdot b$, where $1 \leq i \leq M$. Due to the SNR behavior, the factor which limits the symbol length is random for each sub–carrier. Denote by X_j the random, discrete limiting factor of sub–carrier j. The mean throughput of sub–carrier j will obviously be :

$$D_{\text{Sub. j}} = E(X_j) \cdot \frac{b}{T_s}$$

Recall that we desire a different mean. For the first priority class we desire the mean of the N best sub–carriers out of $3 \cdot N$ sub–carriers in total. Therefore denote by $X_{(j)}$ the limiting factor of the j–th worst sub–carrier towards some wireless terminal. With this notion we desire the following throughput for the first priority class:

$$D_{first} = \left(E\left(X_{(3 \cdot N)}\right) + E\left(X_{(3 \cdot N-1)}\right) + \ldots + E\left(X_{(2 \cdot N+1)}\right) \right) \cdot \frac{b}{T_s} = \sum_{j=2 \cdot N+1}^{3 \cdot N} E\left(X_{(j)}\right) \cdot \frac{b}{T_s} \tag{3.4}$$

According to this, the mean throughput for the second priority class is:

$$D_{second} = \left(E\left(X_{(2\cdot N)}\right) + E\left(X_{(2\cdot N-1)}\right) + \ldots + E\left(X_{(N+1)}\right)\right) \cdot \frac{b}{T_s} = \sum_{j=N+1}^{2\cdot N} E\left(X_{(j)}\right) \cdot \frac{b}{T_s} \tag{3.5}$$

Note that the second priority class chooses out of $2 \cdot N$ sub–carriers the N best ones. The throughput for priority class three is again similar to the throughput of the static sub–carrier assignment scheme (Equation 3.2).

$$D_{third} = \sum_{j=1}^{M} \left(\frac{1}{M} \cdot \frac{j}{M}\right) \cdot \frac{n \cdot b}{T_s}$$

Let us consider the mean throughput of the j–th worst sub–carrier in general, therefore let us consider Expression 3.6.

$$D_{\text{j–th worst Sub.}} = E\left(X_{(j)}\right) \cdot \frac{b}{T_s} \tag{3.6}$$

The random variable $X_{(j)}$ is discrete and will take one of the values $\frac{i}{M}$, where $1 \leq i \leq M$. Since X_j is the limiting factor of the j–th worst sub–carrier , the limiting factors of $j < l \leq 3 \cdot N$ will be $X_{(l)} \geq X_{(J)}$, meaning that the related sub–carriers are better or at least equal. Obviously, due to the discrete character of $X_{(j)}$, we can extend Equation 3.6 and obtain Equation 3.7.

$$D_{\text{j–th worst Sub.}} = E\left(X_{(j)}\right) \cdot \frac{b}{T_s} = \sum_{i=1}^{M} \left(\frac{i}{M} \cdot P\left(X_{(j)} = \frac{i}{M}\right)\right) \frac{b}{T_s} \tag{3.7}$$

Here $P\left(X_{(j)} = \frac{i}{M}\right)$ represents the probability that the j–th worst sub–carrier will be in a SNR state such that the limiting factor of the symbol length is $\frac{i}{M}$. The probability $P\left(X_{(j)} = \frac{i}{M}\right)$ is not equal to p_i, which was introduced in Section 2. The difference is that $P\left(X_{(j)} = \frac{i}{M}\right)$ represents the probability of a certain state for the j–th worst sub–carrier whereas p_i represents the probability of a certain state of a sub–carrier in general. Obviously, $P\left(X_{(j)} = \frac{i}{M}\right)$ is dependent on the number of other sub–carriers in the cell, here $3 \cdot N$.

In order to obtain now $P\left(X_{(j)} = \frac{i}{M}\right)$, we will apply some results from order statistics to this problem. From [1] we know the following. Let X_1, ...,X_Z be independent and identically distributed continuous random variables with probability distribution $F(x) = P(X_j <= x)$. If we let $X_{(j)}$ be the j–th smallest of these random variables (according to the notion of above), then we obtain for the probability that $X_{(j)}$ is less or equal to a certain value x:

$$P\left(X_{(j)} <= x\right) = \sum_{k=j}^{Z} \binom{Z}{j} \cdot (F(x))^k \cdot (1 - F(x))^{Z-k} =$$

$$\sum_{k=j}^{Z} \binom{Z}{j} \cdot (P(X_j <= x))^k \cdot (P(X_j > x))^{Z-k} \tag{3.8}$$

Equation 3.8 gives us the shown probability for continuous random variables. In our case, we have discrete random variables, which can be in one of M states. Therefore we need equation 3.8 applied to discrete random variables. The change is quite straightforward, if we consider the following relationship of our uniformly distributed discrete random variables:

$$P\left(X_j <= \frac{i}{M}\right) = \frac{i}{M}, \qquad P\left(X_j > \frac{i}{M}\right) = \frac{M-i}{M} \tag{3.9}$$

With this we obtain a discrete form of Equation 3.8, where $X_{(j)}$ denotes now the j–th smallest limiting factor of the j–th worst sub–carrier out of $Z = S$ total sub–carriers:

$$P\left(X_{(j)} <= \frac{i}{M}\right) = \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\frac{i}{M}\right)^k \cdot \left(\frac{M-i}{M}\right)^{S-k} \tag{3.10}$$

Equation 3.10 does not provide us with the desired probability of $P\left(X_{(j)} = \frac{i}{M}\right)$. In order to obtain this desired probability, we do the following:

$$\begin{aligned} P\left(X_{(j)} = \frac{i}{M}\right) = P\left(X_{(j)} <= \frac{i}{M}\right) - P\left(X_{(j)} <= \frac{i-1}{M}\right) = \\ \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\frac{i}{M}\right)^k \cdot \left(\frac{M-i}{M}\right)^{S-k} - \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\frac{i-1}{M}\right)^k \cdot \left(\frac{M-i+1}{M}\right)^{S-k} = \\ \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\left(\frac{i}{M}\right)^k \cdot \left(\frac{M-i}{M}\right)^{S-k} - \left(\frac{i-1}{M}\right)^k \cdot \left(\frac{M-i+1}{M}\right)^{S-k}\right) \end{aligned} \tag{3.11}$$

Equation 3.11 provides us with the probability that the j–th worst discrete random variable out of S independent and identically distributed random variables will be equal to the value $\frac{i}{M}$. Therefore we can go on now and obtain the mean value of the j–th smallest discrete random variable by applying Equation 3.11 to Equation 3.7. We obtain:

$$\begin{aligned} E\left(X_{(j)}\right) = \sum_{i=1}^{M} \left(\frac{i}{M} \cdot P\left(X_{(j)} = \frac{i}{M}\right)\right) = \\ \sum_{i=1}^{M} \frac{i}{M} \cdot \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\left(\frac{i}{M}\right)^k \cdot \left(\frac{M-i}{M}\right)^{S-k} - \left(\frac{i-1}{M}\right)^k \cdot \left(\frac{M-i+1}{M}\right)^{S-k}\right) \end{aligned} \tag{3.12}$$

In order to obtain now the mean throughput values for priority class one, we simply use Equation 3.12 in Equation 3.4 where $S = 3 \cdot N$ (note that we set $J = 3$). In order to obtain the mean throughput values for priority class two, we do the same with Equation 3.4 but use for instead $S = 2 \cdot N$. With this we obtain the desired mean throughput value for each wireless terminal as described in Equation 3.3.

One of the already discussed features of the Rotating Sub–carrier Space Algorithm in [7] is the behavior of the simple RSSA but also of the advanced RSSA, that the mean throughput found for the case of three wireless terminals within a cell will increase if the number of wireless terminals increases. Also if the number of available sub–carriers increases, the throughput will increase. The same applies to the simple RSSA in the case of the M–ary sub–carrier state model. The reason is the same as in [7]. If for example the number of wireless terminals increases while the number of available sub–carriers increases accordingly, the probability that the wireless terminal in the first priority class will receive the fair amount of quite good sub–carriers will increase, sine there are more sub–carriers available in total. Therefore a nice feature of the simple RSSA but also of the following advanced versions of the RSSA is, that its throughput will increase if the number of wireless terminals of the number of sub–carriers available increases. This is also the reason for continueing to investigate the case of three wireless terminals, since throughput results can only ioncrease.

3.3 Example Setting and first Comparison of the Policies

To give an impression of the efficiency ratio between the two scheduling policies, let us present here some numerical results. Let us set $S = 12$, $J = 3$, $N = 4$, $b = 8$ bits and in addition $M = 8$. With these values we obtain the following results for the SSA:

$$D_{\text{M–ary uniform SSA}} = \sum_{i=1}^{8} \left(\frac{1}{8} \cdot \frac{i}{8} \right) \cdot \frac{4 \cdot b}{T_s} = 0.5625 \cdot \frac{4 \cdot b}{T_s} = 18 \frac{\text{bits}}{T_s}$$

For the simple RSSA algorithm we obtain the following results:

$$D_{first} = 0.8665 \cdot \tfrac{4 \cdot b}{T_s} \qquad D_{second} = 0.7821 \cdot \tfrac{4 \cdot b}{T_s} \qquad D_{third} = 0.5625 \cdot \tfrac{4 \cdot b}{T_s}$$

$$\Rightarrow D_{\text{M–ary uniform SRSSA}} = 0.737 \cdot \tfrac{4 \cdot b}{T_s} = 23.58 \frac{\text{bits}}{T_s}$$

This corresponds to an increase of 31 % for the simple RSSA compared to the static assignment scheme. In contrast to the case of the simple RSSA in the binary sub–carrier state model [7], the throughput now might be improved by increasing the throughput of all three priority classes. This represents a fundamental difference of the M–ary sub–carrier state model compared to the binary sub–carrier state model.

In Figure 3.1 the distribution plots of individual sub–carrier weights is given as assigned to each priority class. By individual sub–carrier weight we mean the sub–carrier weight as seen by a single wireless terminal or a priority class. Therefore if a wireless terminal may receive $\frac{q \cdot b}{M}$ bits on a certain sub–carrier for some time unit, the individual sub–carrier weight as seen by this wireless terminal is $\frac{q}{M}$. As we can see, although the original distribution of the individual sub–carrier weights is uniform (corresponding to the uniform SNR distribution assumed), only for priority class three the individual sub–carrier weights are also uniformly distributed. The distributions for the first and second priority classes are as expected much more dense for higher sub–carrier weights. Also, when comparing the distributions between priority class one and priority class two we observe, that the distribution for priority class one lies above the distribution of priority class two for higher sub–carrier weights. This corresponds to the throughput results, since priority class one has a much higher mean throughput per sub–carrier than priority class two.

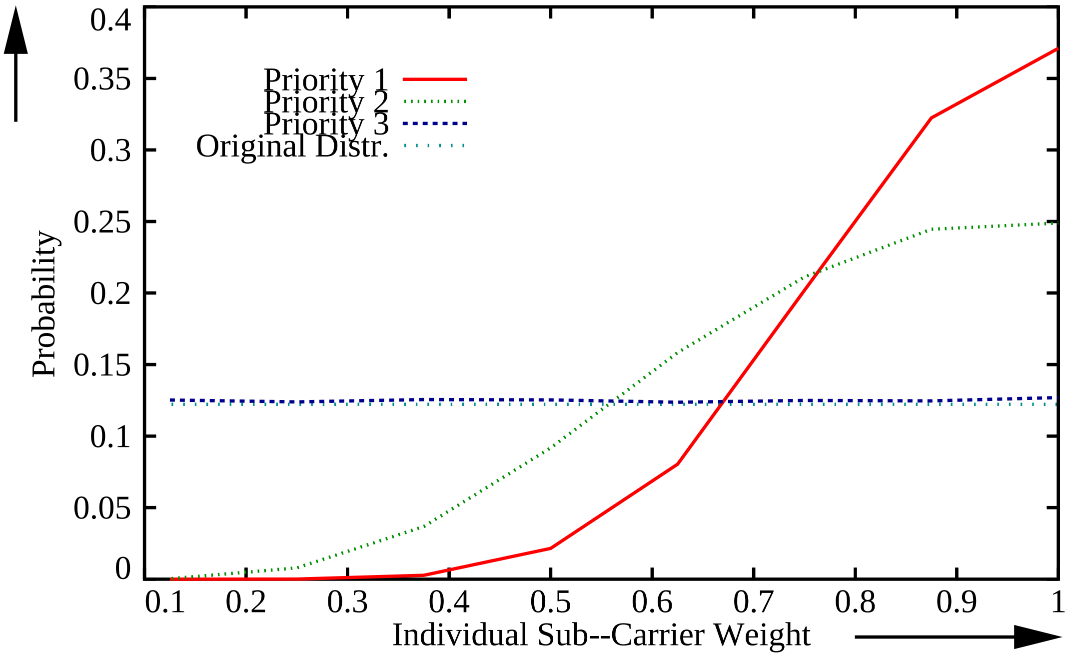

Figure 3.1: Distribution of the Individual Sub–Carrier Weights assigned to different Priority Classes for $J = 3$, $N = 4$, $S = 12$, and $M = 8$

Another interesting feature of the simple RSSA scheduling policy is captured in Figure 3.2. It shows the distributions of total sub–carrier weights as assigned to each priority classes. By the total sub–carrier weight we mean the sum of the individual sub–carrier weights. Therefore, if a sub–carrier is able to convey $\frac{r \cdot b}{M}$ bits to one wireless terminal, while being able to transmit $\frac{s \cdot b}{M}$ bits to a second wireless terminal and being able to transmit $\frac{t \cdot b}{M}$ bits to a third wireless terminal, then the sub–carrier has a total weight of $\frac{r+s+t}{M}$. The Figure also includes the original distribution of sub–carrier weights corresponding to the assumed statistical model for the sub–carriers. As we can see, all priority classes have the same shape of distribution, which corresponds to a Gaussian distribution. But the mean values (corresponding roughly to the maximum value of the distributions) are different, therefore creating shifted versions of the original distribution. For priority class one the distribution is shifted at most to the right, whereas for priority class three the distribution is shifted at most to the left. This corresponds of course to the fact, that priority class one obtains the most high quality sub–carriers. If the individual sub–carrier weight received by the access point (and therefore the sub–carriers data rate per wireless terminal) is higher, the total sub–carrier weight assigned will also be higher and vice versa.

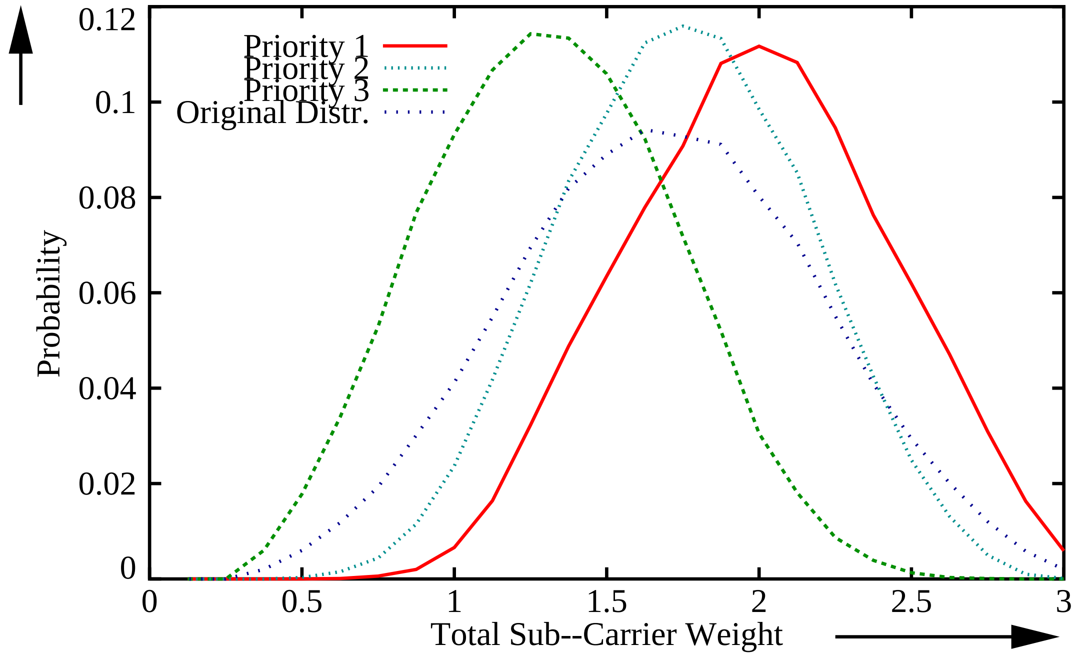

Figure 3.2: Distribution of the Total Sub–Carrier Weights assigned to different Priority Classes for $J = 3$, $N = 4$, $S = 12$, and $M = 8$

3.4 Advancing the Simple RSSA

For the following discussion, we will keep up the assumption of $J = 3$. Mainly, this is for illustration purposes. However, as mentioned before, this is also because for more than three wireless terminals in the cell throughput results will increase. Therefore three wireless terminals describes a lower limit for the throughput behavior of the different RSSA versions.

As with the binary sub–carrier state model (refer to [7]), the mean throughput of the simple RSSA can be improved. Again this can be achieved by introducing additional policies regarding the choosing of sub–carriers for different priority classes by the access point. We will refer to this additional functions as choosing function. From the values of Section 3.3 it seems at the first glance as if the throughput not only of the third priority class may be improved but also of the second and first priority class. Since the mean throughput values of the second and third class are much smaller compared to the first priority class, we will focus primarily on improving the throughput for the second and third priority class. In contrast to the binary sub–carrier state model, any improvement regarding the mean throughput values of the lower two priority classes will probably include a reduction of the mean throughput of the highest priority class. The reason for this is a consequence of the different sub–carrier state model assumed here. In the case of the binary sub–carrier state model the highest priority class was assigned sub–carriers which had at best a low total weight. There was no additional constraint to take into account since all considered sub–carriers were in a good state and had therefore the same individual weight. In the case of the M–ary sub–carrier state model, this unique quality of sub–carriers is not valid anymore. Therefore a choosing function has to evaluate the case that a sub–carrier with a high individual weight towards the actual wireless terminal in priority class one has also a high total weight, whereas the next best sub–carrier towards this wireless terminal has a lower total weight but also a significant lower individual weight. It turns out to be quite difficult for the choosing function to maximize the overall throughput per wireless terminal.

In the following we present three advanced RSSA scheduling policies, from which two of them rely on the same idea. For the choosing function two alternatives may be imagined. First of all the choosing function could consider for each priority class the quality of the actual sub–carrier towards the next priority class. Such a choosing function would decide on a fraction of given sub–carrier state information. Such a choosing function would only take into account sub–carrier states towards the wireless terminal in priority class two when considering the sub–carrier assignments for priority class one. According to this for priority class two the choosing function would include sub–carrier state information about priority class three. Through such a partial information function the throughput of priority class one might be slightly decreased in order to assign a better throughput to priority class two. Accordingly, the throughput of priority class two might be slightly decreased for a better throughput of priority class three.

As a second approach the choosing function could consider the complete sub–carrier state information given. In such a case the total given information would be used in order to assign sub–carriers to priority classes. Therefore a choosing function would take into account the total sub–carrier weight when assigning sub–carriers to the wireless terminal actually being in class one, for example. This corresponds to the way the advanced RSSA works in the binary sub–carrier state model.

As first choosing function we introduce one according to the first principle. In detail the choosing function forces the access point to only assign a sub–carrier to a priority class wireless terminal if the sub–carriers individual weight towards the actual considered wireless terminal is better than the sub–carriers individual weight towards the next priority class wireless terminal. In practice the access point would consider when assigning sub–carriers for priority class one the individual sub–carrier weight towards priority class two and so on. Let us denote this overall scheduling policy as advanced RSSA 1.

As a second choosing function for the RSSA following the first mentioned idea we changed the advanced RSSA 1 such that the access point assigns sub–carriers even in the case that the next priority class wireless terminal sub–carriers individual weight is as good as the actual one. Therefore, if both priority class one and priority class two for example have the same sub–carrier individual weight, priority class one would get the sub–carrier assigned, in contrast to the advanced RSSA 1. This scheduling policy is just a minor modification of the advanced RSSA 1, but still let us denote it by advanced RSSA 2.

The third choosing function follows the second mentioned principle. In this case the access point considers the mean individual weight results from the total sub–carriers weight divided by the number of wireless terminals in the cell. If the mean value is lower than the single sub–carrier weight towards the actual considered priority class then the sub–carrier will be assign to the wireless terminal. If this is not the case (and therefore the mean value is bigger or equal) the sub–carrier will not be assigned to the wireless terminal, since there must be another wireless terminal for which the sub–carriers individual weight is higher. This scheduling policy will be denoted as advanced RSSA 3.

Algorithm	Throughput Ratio Overall	Priority Class One
Simple RSSA	0.737	0.8665
Advanced RSSA 1	0.7406	0.7843
Advanced RSSA 2	0.7524	0.8386
Advanced RSSA 3	0.7470	0.8266

Table 3.1: Detailed Throughput Results of all RSSA version for the uniform M–ary Sub–Carrier State Model, Part 1

Algorithm	Pr. Class Two	Pr. Class Three
Simple RSSA	0.7821	0.5625
Advanced RSSA 1	0.7220	0.7155
Advanced RSSA 2	0.7601	0.6587
Advanced RSSA 3	0.7478	0.6667

Table 3.2: Detailed Throughput Results of all RSSA version for the uniform M–ary Sub–Carrier State Model, Part 2

As with the binary advanced RSSA version, it turns out to be quite difficult to obtain mean throughput values by analysis. Therefore in order to learn about the efficiency of the advanced RSSA versions we have to rely on simulations. Table 3.1 holds the mean throughput results for all three advanced RSSA versions. The numbers given are the ratio between the mean throughput and the throughput in the case that each sub–carrier is in the best available quality state for every wireless terminal , offering therefore a throughput of $b\frac{\text{bits}}{T_s}$. As we can see from the results the advanced RSSA 1 implies a throughput increase of roughly $\frac{1}{2}$ percent, whereas advanced RSSA 2 implies a throughput increase of 2 percent and advanced RSSA 3 implies an increase by 1.3 percent. Surprisingly the choosing function principle only using partial sub–carrier state information outperforms the principle utilizing the full sub–carrier state information. As with the mean throughput, the advanced RSSA 2 has the best throughput for the upper two priority classes by reducing the throughput of priority class three at most. Interestingly the choosing function version that achieves a relative uniform throughput between the three priority classes, which is advanced RSSA 1, achieves the worst throughput of all advanced RSSA versions. As mentioned before, in order to increase the throughput of lower priority classes the throughput of upper priority classes has to be decreased due to the statistical nature of the sub–carrier state model assumed here. This is shown very clearly from the numerical results given in Table 3.1 and 3.2.

At next we also show the distribution plots for all three advanced RSSA versions of the individual sub–carrier weights as assigned to each priority class. Also we show the distribution plots of the total sub–carrier weights as assigned to each priority classes. For both distribution plots we also present the original distributions resulting from the assumed statistical sub–carrier model. Let us discuss these by comparing the plots for each advanced version with the plots of the simple RSSA. We start with the advanced RSSA 1.

As already stated is the advanced RSSA 1 the version with the lowest throughput increase

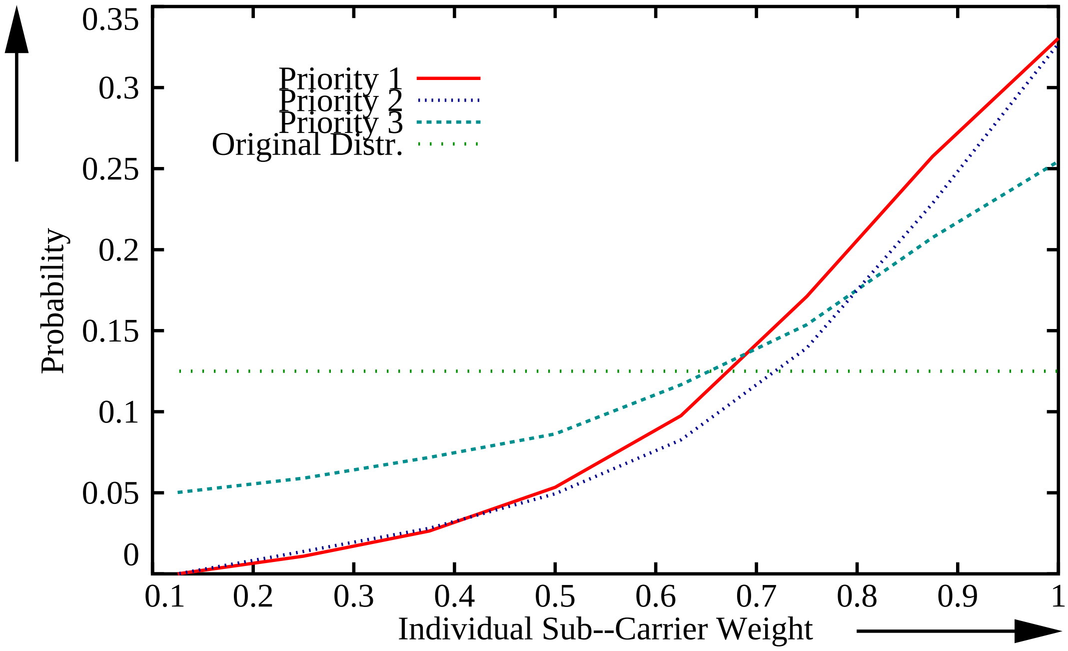

Figure 3.3: Distribution of the Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 1 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

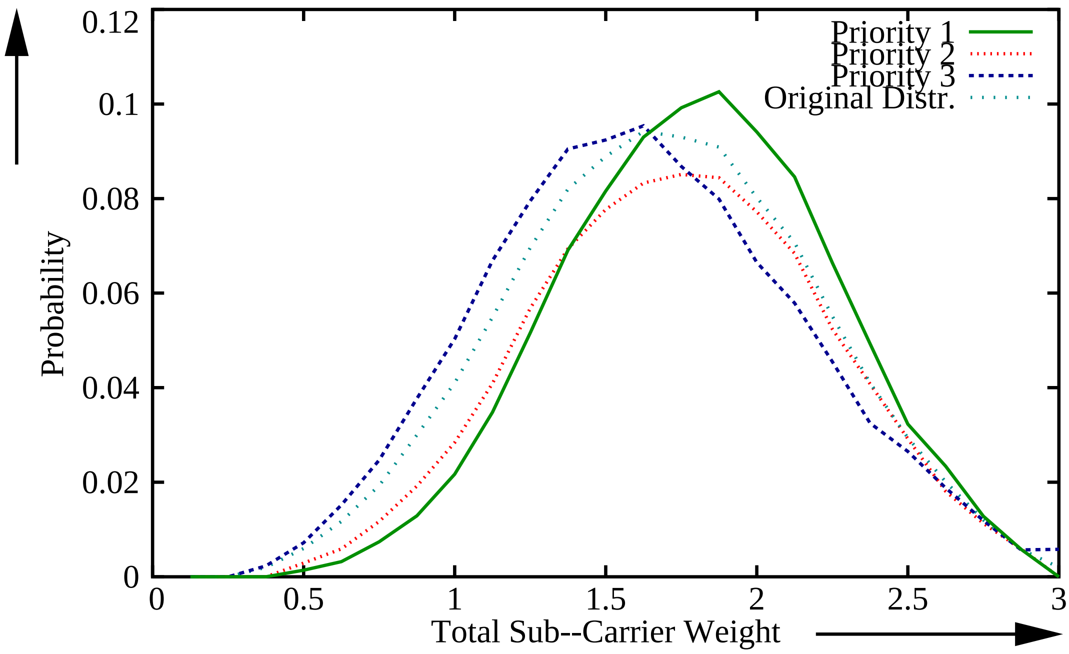

Figure 3.4: Distribution of the Total Sub–Carrier Weights per Priority Class for the advanced RSSA 1 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

of $\frac{1}{2}$ %. Now, if we compare the distribution plots for the individual sub–carrier weight of the simple RSSA (Figure 3.1) with the one of the advanced RSSA 1 (Figure 3.3) we can observe, that the curves for the advanced RSSA 1 are much more similar to each other than in the case of the simple RSSA. This corresponds to the observation, that the throughput values in case of the advanced RSSA 1 have the smallest spread. While the throughput for the upper two priority classes is reduced, it is increased for the lowest priority class. Therefore, the advanced RSSA 1 is the least discriminating version of all RSSA versions. The same observation made for the individual sub–carrier weights can be found for the total sub–carrier weight, when comparing the plot of the simple RSSA (Figure 3.2) with the plot of the advanced RSSA 1 (Figure 3.4). Again the distributions for the different priority classes are almost identical for the advanced RSSA 1, which is in contrast to the distributions of the simple RSSA.

The advanced RSSA 2 is the version with the best throughput increase. However, it has also the largest throughput spread between the different priority classes, except for the simple RSSA. When comparing the distributions of the individual sub–carrier weight of the simple RSSA (Figure3.1) with the one of the advanced RSSA 2 (Figure 3.5) we can see, that the third priority class has no longer a uniform shape. Instead it is also slightly increasing. However, the spread between the graph of the first and last priority class is much larger as in the case of the advanced RSSA 1. Whe comparing the plots for the total sub–carrier weight of the simple RSSA (Figure 3.2) with the plots of the advanced RSSA 2 (Figure 3.6), we observe that the peak values for all three priority classes have stayed the same, but the height of the peaks is different. In the case of the simple RSSA all peaks had the same height, whereas in the case of the advanced RSSA 2 priority class three and one have the same height but priority class two has a lower height. On the other hand, the width of the graph of priority class two is much wider than in the case of the other two priority classes.

While the advanced version 1 and 2 only took the sub–carrier state of the next lowest priority class into account, advanced RSSA 3 is the version, which considers the state of all priority classes of a sub–carrier when assigning a sub–carrier to a wireless terminal. Interestingly, the throughput increase is slightly lower than in the case of the advanced RSSA 2. When comparing the plots of the individual sub–carrier weights of the simple RSSA (Figure3.1) with the one of the advanced RSSA 3 (Figure 3.7), we observe again that the graph of the third priority class is no longer uniform. The graph of the first priority class is slightly decreased as well as the graph of the second priority class. Interesting is the distribution of the total sub–carrier weight for the advanced RSSA 3 (Figure 3.8) when comparing it to the plot of the simple RSSA (Figure 3.2). In the case of the simple RSSA the plot is a lot smoother than in case of the advanced RSSA 3.

3.5 Upper Throughput Limits of Scheduling Policies for the M–ary Sub–Carrier State Model with an uniform Distribution

As in [7] we are again interested in finding an upper throughput bound for the used statistical sub–carrier model. As in [7], we used in section 3.4 and section 3.3 as performance measure the ratio between the absolute throughput for the scheduling policy and the throughput achievable in the case that all sub–carriers are always in the best SNR zone (and therefore

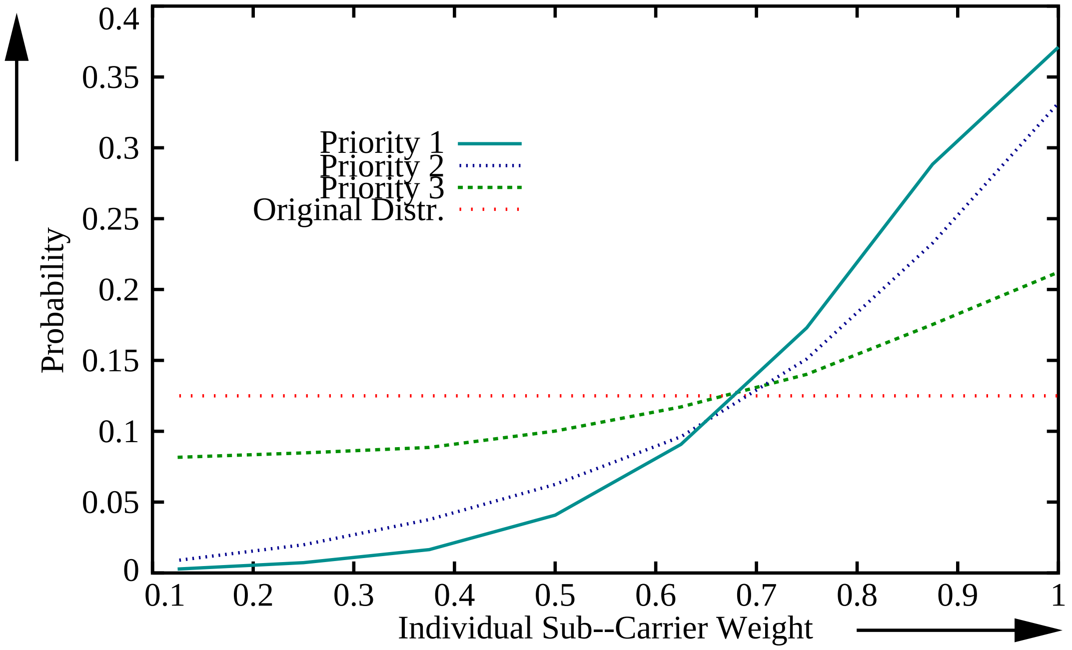

Figure 3.5: Distribution of the Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 2 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

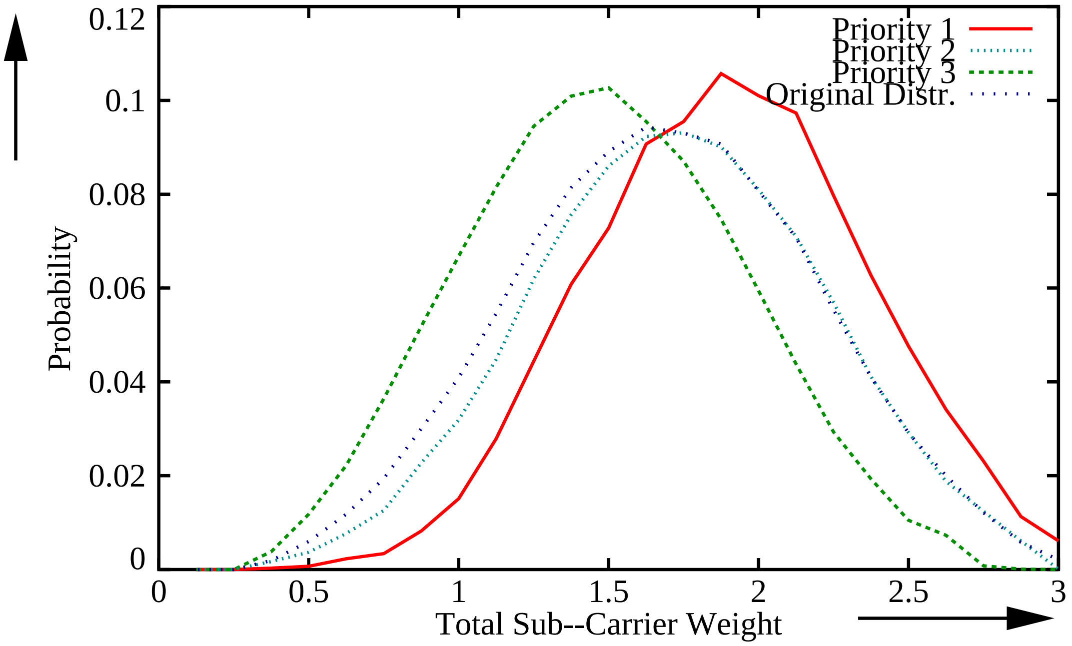

Figure 3.6: Distribution of the Total Sub–Carrier Weights per Priority Class for the advanced RSSA 2 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

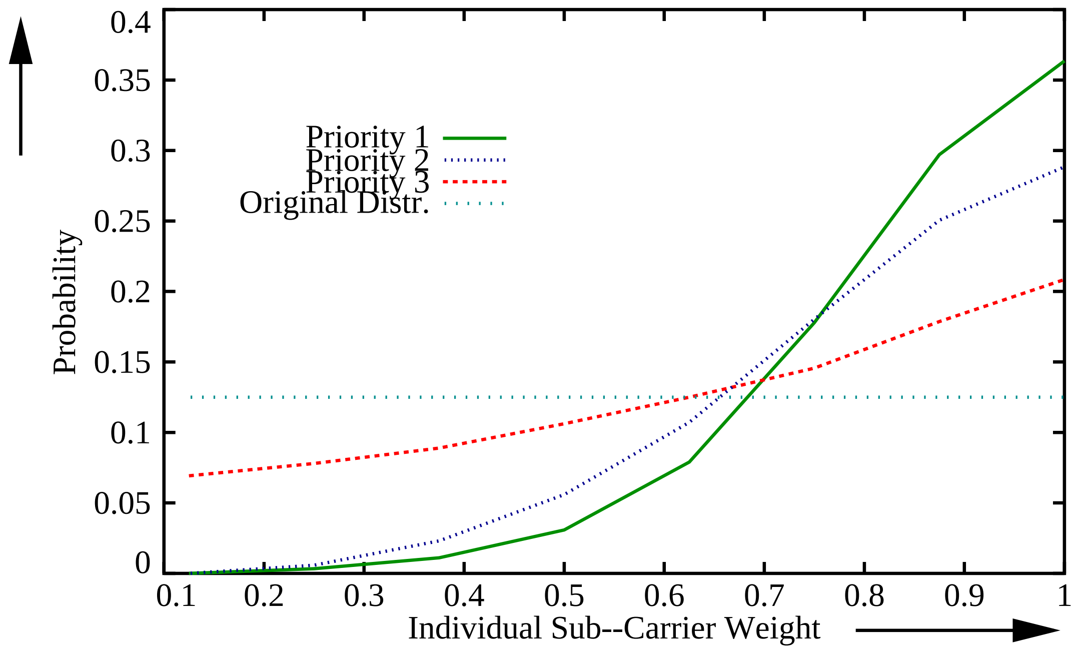

Figure 3.7: Distribution of the Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 3 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

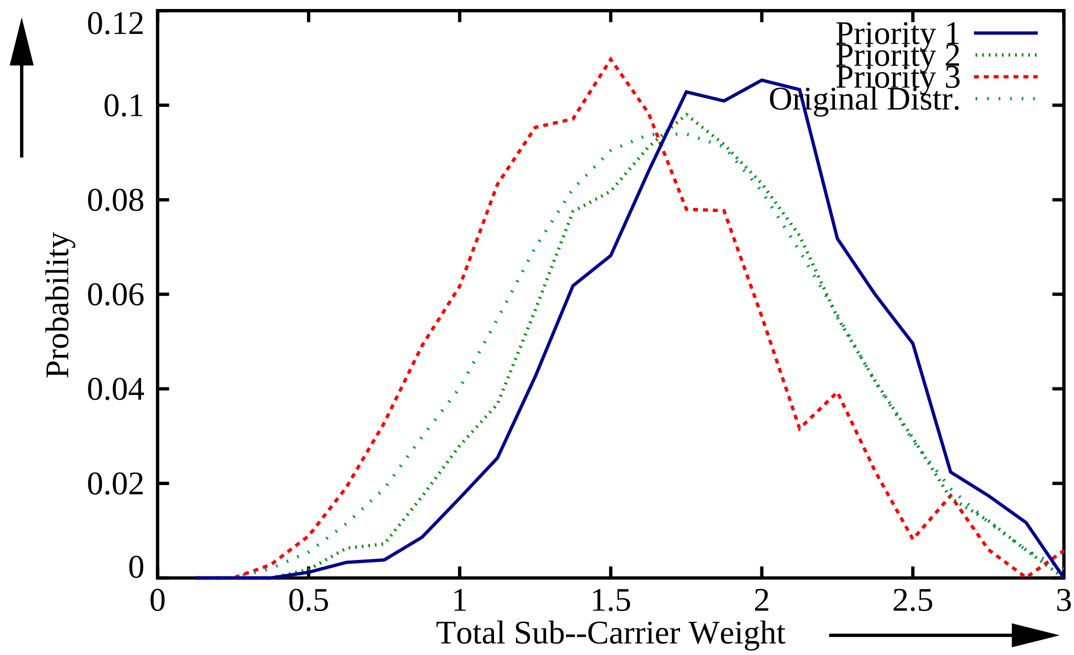

Figure 3.8: Distribution of the Total Sub–Carrier Weights per Priority Class for the advanced RSSA 3 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

enable the usage of the modulation scheme conveying the most information per symbol). But this ratio will not tell us, how good the scheduling policy is compared to the maximal throughput achievable within the M–ary sub–carrier state model. In other words, from the actual point of view it is difficult to judge how much improvement might be achieved by a fourth advanced RSSA version. In this section we will focus on this question.

Two optimal policies will be introduced here. The first policy will not consider fairness to be an important constraint on a time scale of the time span T_s. Until now, fairness was always given by assigning each wireless terminal N sub–carrier per time unit. This constraint is now not a condition any more. If we do not take this constraint into account, clearly an optimal policy would be to always assign a sub–carrier to the wireless terminal for which the sub–carrier is in the best state. In such a policy short–term fairness is not present, since for some symbol period maybe all sub–carriers are in the best state towards one wireless terminal. For example this could be the case if a wireless terminal is located very close to an access point. However, due the independence assumptions stated in [7], long–term fairness can be guaranteed. This means, due to the statistical model each wireless terminal will once in a while get more than N sub–carriers assigned. Since there are no correlation assumptions in the statistical model of the sub–carrier behavior, over a long time scale this will cancel out, since it will happen to all wireless terminals. We will refer to this policy as long–term fair optimal policy.

The second policy keeps the constraint of short–term fairness as used up until now. Therefore the constraint is still valid that every wireless terminal will obtain N sub–carriers per time unit. But now complexity is increased to a maximum. Every possible assignment combination of the S total sub–carriers in the cell and the J wireless terminals are considered with respect to the fairness constraint. Therefore for every combination of N assigned sub–carriers to J wireless terminals in the cell the mean throughput per wireless terminal is calculated and the one with the maximum throughput is picked.. This policy owns the highest possible complexity, but will also achieve the maximal throughput for policies guaranteeing short–term fairness. We will refer to this policy as short–term fair optimal policy.

Both policies were simulated with the same variable settings as used in section 3.3. We obtained the following results shown Equation 3.13 and 3.14:

$$D_{\text{long–term fair optimal}} = 0.809 \cdot \frac{n \cdot b}{T_s} = 25.88 \frac{\text{bits}}{T_s} \tag{3.13}$$

$$D_{\text{short–term fair optimal}} = 0.7953 \cdot \frac{n \cdot b}{T_s} = 25.45 \frac{\text{bits}}{T_s} \tag{3.14}$$

If we build now the ratio between the two optimal policies introduced here and the results obtained so far for the SSA, the simple RSSA and the advanced RSSA scheduling policies, we obtain the results shown in Table 3.3 and 3.4 :

As we can see in Tables 3.3 and 3.4 does the advanced RSSA 2 scheduling policy achieve 93 % of the upper throughput bound of the long–term optimal policy and 94 % of the short–term optimal policy. Here we observe an interesting trade-off between throughput, fairness and complexity. Advanced RSSA 2 achieves a lower throughput compared to the long–term and short–term optimal policies but has also either less complexity or provides more fair assignments of sub–carriers. If we neglect the constraint on complexity or on fairness we

Algorithm	Normalized Throughput (long–term fair optimal)
SSA	0.695
Simple RSSA	0.911
Advanced RSSA 1	0.915
Advanced RSSA 2	0.93
Advanced RSSA 3	0.923

Table 3.3: Normalized Throughput Results related to the long–term optimal scheduling policies for the M–ary sub–carrier state model with a uniform SNR distribution

Algorithm	Normalized Throughput (short–term fair optimal)
SSA	0.707
Simple RSSA	0.926
Advanced RSSA 1	0.931
Advanced RSSA 2	0.946
Advanced RSSA 3	0.939

Table 3.4: Normalized Throughput Results related to the short–term fair optimal scheduling policies for the M–ary sub–carrier state model with a uniform SNR distribution

can achieve a better throughput. Still the six percent (respectively five percent) throughput difference is quite a good result, if we consider the provided fairness and the low complexity of the advanced RSSA 2.

Chapter 4

Scheduling Policies for the M–ary Sub–Carrier State Model with an Gaussian SNR Distribution

In this section we discuss all results related to the M–ary sub–carrier state model with a Gaussian SNR distribution. At first we present analytical results on the static assignment scheme. Then we show results for a simple dynamic scheduling policy. This policy can be advanced in order to increase the throughput. This is done in the successing section. At last, we also discuss theoretical upper bounds for this model and compare the presented scheduling policies to the upper bounds.

4.1 Static Sub–Carrier Assignments – SSA

Since we only changed the SNR distribution from uniform to Gaussian, the only thing differing in our analytical derivation of the mean throughput per wireless terminal is the probability that a certain modulation type will be used (p_i). Referring to the formulas in Section 2, we obtain for the mean throughput per wireless terminal:

$$D_{\text{M–ary Gaussian SSA}} = \sum_{i=1}^{M} \left(p_i \cdot \frac{i}{M} \right) \cdot \frac{n \cdot b}{T_s} =$$

$$\sum_{i=1}^{M} \left(\int_{i_{low}}^{i_{up}} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x - SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx \cdot \frac{i}{M} \right) \cdot \frac{n \cdot b}{T_s} \tag{4.1}$$

4.2 Simple Rotating Sub–Carrier Space Algorithm – Simple RSSA

As with the SSA scheduling policy, the derivation of the mean throughput for the simple RSSA per wireless terminal has only to be adjusted to the probability that a certain SNR value lies within zone i. Of course Equations 3.3, 3.4 and 3.5 still hold for our new sub–carrier state model. For the throughput of the third priority class, we refer to Equation 4.1. In order to obtain now the mean throughput values of the j–th worst sub–carrier chosen by a priority class (Equation 3.7), we have to modify Equation 3.10 by adjusting Equation 3.9 to the new statistical model assumed. From the Gaussian distribution the adjusted Equation 3.9 takes the form:

$$P\left(X_j <= \frac{i}{M}\right) = \int_{-\infty}^{i_{up}} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x-SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx$$

$$P\left(X_j > \frac{i}{M}\right) = 1 - P\left(X_j <= \frac{i}{M}\right) = \int_{i_{up}}^{\infty} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x-SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx \qquad (4.2)$$

Using Equation 4.2 in Equation 3.10 yields:

$$P(X_{(j)} <= \frac{i}{M}) = \sum_{k=j}^{S} \binom{S}{k} \cdot \left(\int_{-\infty}^{i_{up}} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x-SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx \right)^{k} \cdot \left(\int_{i_{up}}^{\infty} \frac{1}{\sqrt{2\pi \cdot SNR_{Var}}} \cdot e^{-\frac{(x-SNR_{Mean})^2}{2 \cdot SNR_{Var}}} \cdot dx \right)^{I-k} \qquad (4.3)$$

With this we can continue to obtain $P\left(X_{(j)} = \frac{q}{M}\right)$ as in Equation 3.11. This new relationship yields then $E\left(X_{(i)}\right)$ as in Equation 3.12, from which the mean throughput can be obtained.

We already described the behavior of the simple RSSA for the case of more than three wireless terminals in the cell. The result is that the mean throughput per wireless terminal will increase with an increase of either wireless terminals or sub–carriers available in the cell. Refer to 3.2 for more information.

4.3 Example Setting and first Comparison of the Policies

Now let us give some numerical impressions on the formulas found so far. As in the Section 3.3 we will set $S = 12$, $J = 3$, $N = 4$, $b = 8$ bits and $M = 8$. Further for the Gaussian SNR distribution we assume $SNR_{mean} = 16$ dB and $SNR_{var} = 4$ dB. The SNR zone width is 2 dB, the most left zone ends at 10 dB, the most right zone starts at 22 dB. For this setting we obtain a mean throughput per wireless terminal for the SSA scheduling policy of:

$$D_{\text{M–ary Gaussian SSA}} = \sum_{i=1}^{8} \left(p_i \cdot \frac{i}{8}\right) \cdot \frac{4 \cdot b}{T_s} = 0.5625 \cdot \frac{4 \cdot b}{T_s} = 18 \frac{\text{bits}}{T_s}$$

For the simple RSSA algorithm we obtain the following results:

$$D_{first} = 0.8062 \cdot \frac{4 \cdot b}{T_s} \qquad D_{second} = 0.7353 \cdot \frac{4 \cdot b}{T_s} \qquad D_{third} = 0.5625 \cdot \frac{4 \cdot b}{T_s}$$

$$\Rightarrow D_{\text{M–ary Gaussian SRSSA}} = 0.7013 \cdot \frac{4 \cdot b}{T_s} = 22.48 \frac{\text{bits}}{T_s}$$

This corresponds to an increase of 25 % for the simple RSSA scheduling policy compared to the SSA. Again let us consider the distribution of individual sub–carrier weights per priority class and the distribution of total sub–carrier weights per priority class in order to obtain some more information about the efficiency of the algorithm at these changed circumstances.

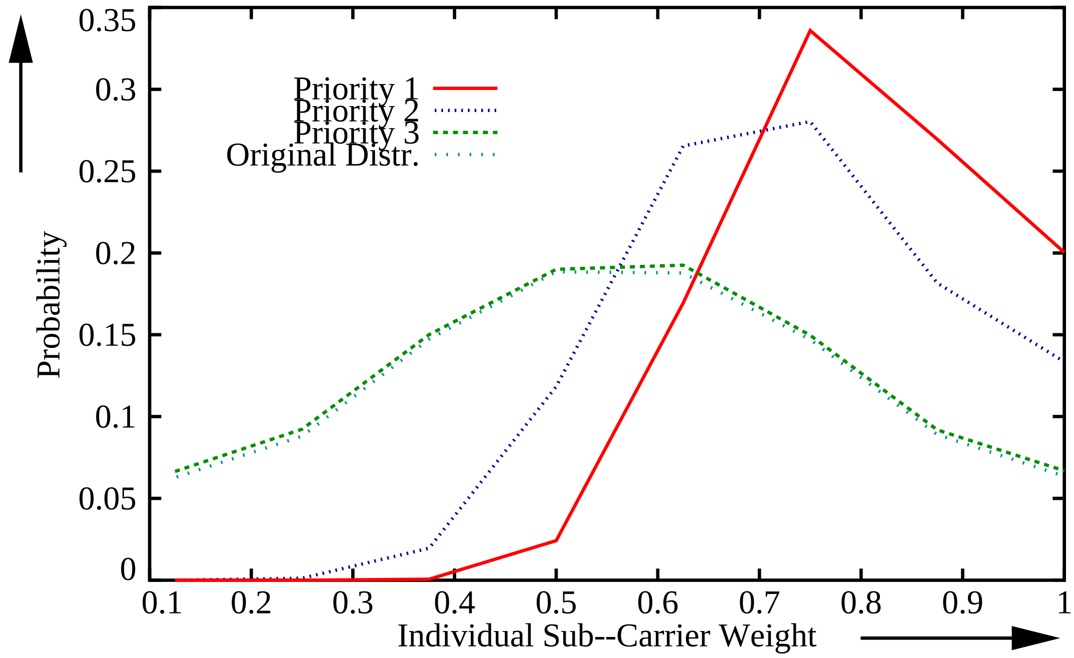

Figure 4.1: Distribution of the Individual Sub–Carrier Weights per Priority Classes for $J = 3$, $N = 4$, $S = 12$, and $M = 8$

As with the sub–carrier model with the uniform distribution (Figure 3.1) we see in Figure 4.1 that the third priority class obtains individual sub–carrier weights as they are originally distributed. For the upper two priority classes the ratios towards each other stay almost the same as in Figure 3.1 but for the highest individual weight value there is a decrease for both priority classes. This is simply due to the situation that these sub–carrier weights are in the Gaussian SNR distribution quite rare and do not occur that often as in the uniform SNR distribution case. For the total sub–carrier weight distribution per priority class as shown in Figure 4.2 we observe the same as in Figure 3.2. Each priority class distribution has the same shape, they have the same height and width. The only difference is that the distributions are shifted towards each other such that the priority class one obtains overall the 'heaviest' total sub–carriers and priority class three obtains the 'lightest' total sub–carriers.

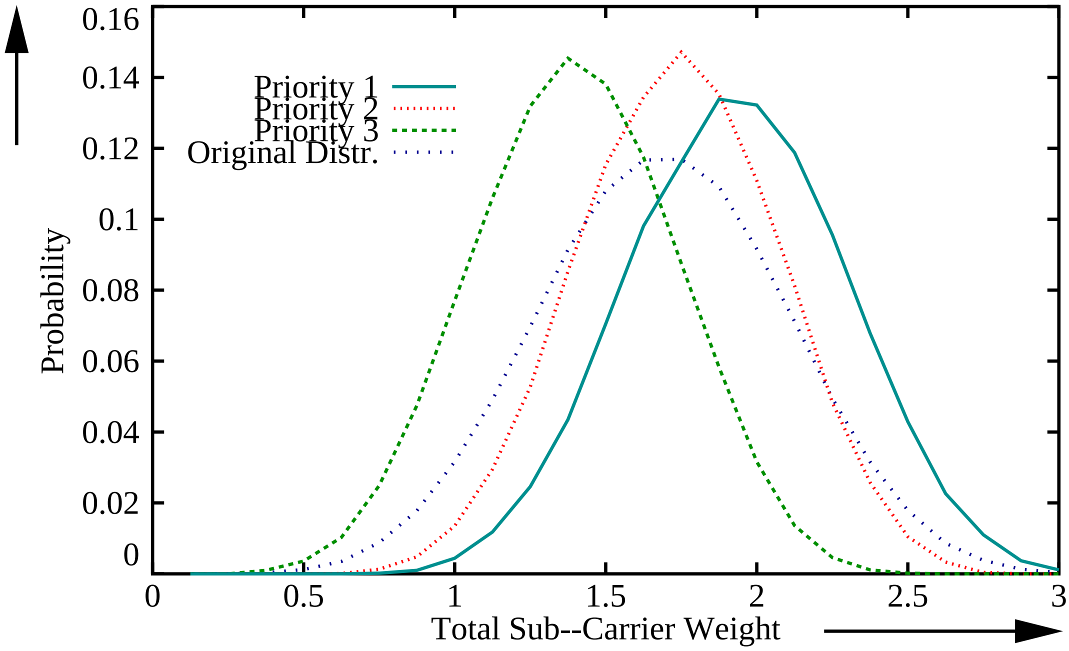

Figure 4.2: Distribution of the Total Sub–Carrier Weights per Priority Classes for $J = 3$, $N = 4$, $S = 12$, and $M = 8$

4.4 Advancing the Simple RSSA

Now let us apply the advanced RSSA versions introduced in Section 3.4 which choose sub–carriers more careful with respect to other priority class wireless terminals. We use again the same values as in the previous Section 4.3. With this we obtain mean throughput values per wireless terminal. In Table 4.1 the ratios between these mean throughput values and the perfect throughput for all sub–carriers always being in the best SNR state are given. Later on we will also come up with the notion of the normalized throughput.

As we can see the mean throughput of the simple RSSA is decreased in the case of the advanced RSSA 1 by 0.3 %, for the advanced RSSA 2 the throughput is increased by 2 % and for the advanced RSSA 3 the throughout is increased by 1 %. Again the advanced RSSA

Algorithm	Throughput Ratio Overall	Pr. Class One
Simple RSSA	0.7013	0.8062
Advanced RSSA 1	0.6987	0.7321
Advanced RSSA 2	0.7158	0.7858
Advanced RSSA 3	0.7082	0.7661

Table 4.1: Detailed Throughput Results of all RSSA version for the Gaussian M–ary Sub–Carrier State Model, Part 1

Algorithm	Pr. Class Two	Pr. Class Three
Simple RSSA	0.7353	0.5625
Advanced RSSA 1	0.6752	0.6889
Advanced RSSA 2	0.7188	0.6428
Advanced RSSA 3	0.6968	0.6616

Table 4.2: Detailed Throughput Results of all RSSA version for the Gaussian M–ary Sub–Carrier State Model, Part 2

1 achieves the most similar mean throughput values for all three priority classes. In fact, we observe that the advanced RSSA 1 decreases the throughput of priority class two below the value of priority class three. This is quite surprising and remains to be explained. One reason for this behavior is that priority class three would have a lower mean throughput compared with priority class two if a fourth priority class would exist and priority class three would have to match its desired sub–carriers with the sub–carriers state behavior towards this fourth priority class. Since there is no fourth priority class, the wireless terminal of priority class three benefits of the quite strict choosing function policy. Priority class two's mean throughput is decreased by this strict choosing function that much that priority class three's mean throughput becomes bigger than that of priority class two. Of course this reveals a rather unfortunate feature of the advanced RSSA 1 scheduling policy. Also, as in the uniform SNR distribution case of Section 3.4, the less strict version of the advanced RSSA 1, the advanced RSSA 2, outperforms all other scheduling policies. Again, it has the biggest mean throughput spread between the throughput values of the different priority classes, beside the spread of the simple RSSA. In contrast to the previous case with the uniform SNR distribution, the advanced RSSA 3 does not achieve such a good percentage increase of the mean throughput (here we only encounter an increase of 1 %, whereas in the uniform case we experienced an increase of 1.3 %).

Now let us continue to observe the results for this different statistical sub–carrier model by discussing the distribution functions for the individual sub–carrier weights and the total sub–carrier weights. Again we will skip through each advanced RSSA version and discuss the plots while comparing them to the plots of the simple RSSA.

If we observe the distribution of the individual sub–carrier weights per priority classes for the advanced RSSA 1 (Figure 4.3) and compare it with the plot for the simple RSSA (Figure 4.1), the throughput decrease of priority class two to the fortune of priority class three can be seen quite clear. Only in the case of the highest sub–carrier weight value, priority class two gets a sub–carrier more often assigned than priority class three. In all other cases priority class three gets more sub–carriers assigned than priority class two, which leads to the effect of the throughput being higher for priority class three than the one of priority class two. Again the advanced RSSA 1 is the policy with the smallest spread between the throughput values of the different priority classes. Therefore the graphs of the single priority classes have quite similar shapes. If we compare the plots for the total sub–carrier weight of the advanced RSSA 1 (Figure 4.4) with the plot of the simple RSSA (Figure 4.2) we observe, that the advanced RSSA 1 scheduling policy moves the different graphs for the priority classes closer together.

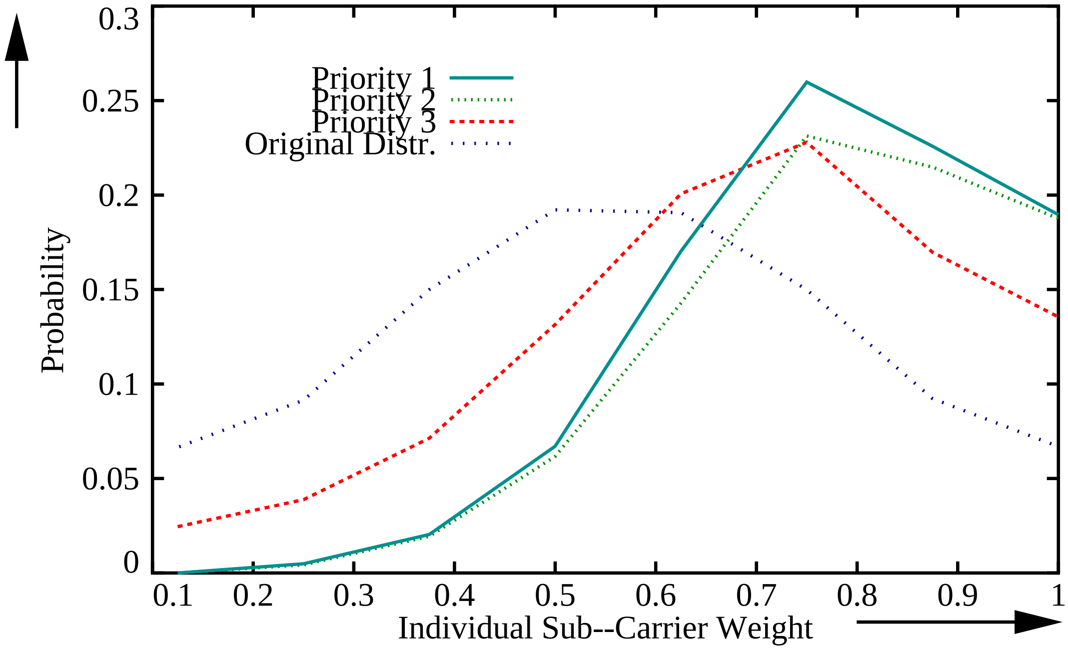

Figure 4.3: Distribution of the Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 1 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

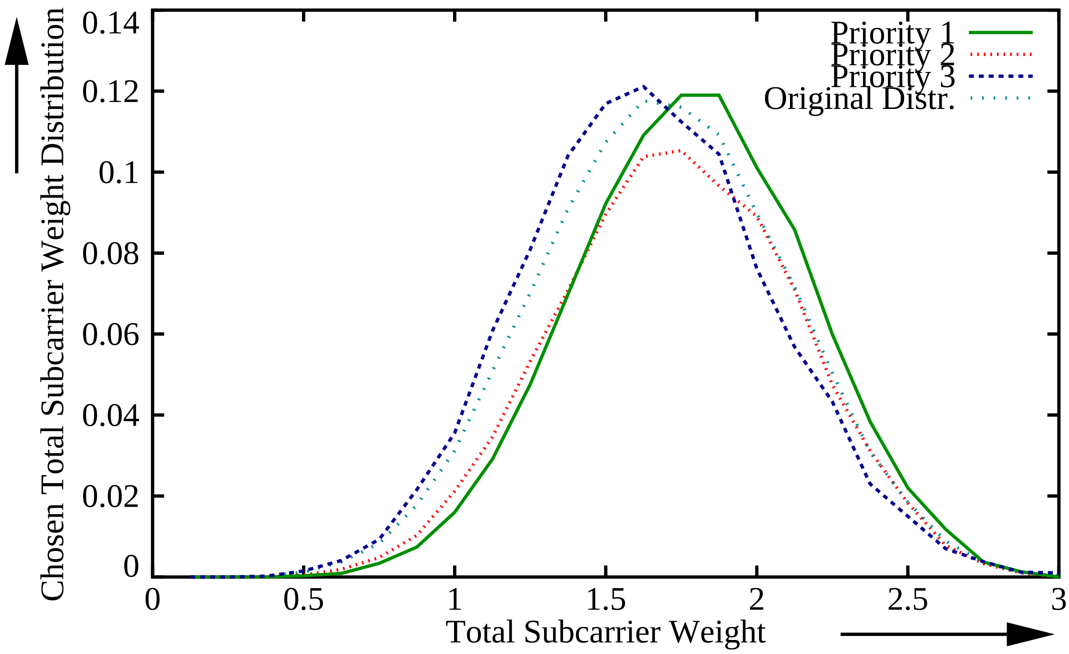

Figure 4.4: Distribution of assigned total Sub–Carrier Weights per Priority Class for the advanced RSSA 1 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

On the other hand the advanced RSSA 1 makes the distribution plot wider than they are for the simple RSSA. This is due to the choosing function of the advanced RSSA 1, which is the most restrictive one by only assigning sub–carriers to a priority class if the sub–carrier SNR state towards the next priority class is worse than the actual considered sub–carrier state.

For the advanced RSSA 2 things stay almost the same as in the uniform distributed SNR case. Again the advanced RSSA 2 is the version with the highest throughput increase and has the biggest throughput spread for the different priority classes. Interesting is for the plot of the individual sub–carrier weight distribution (Figure 4.5), that although the advanced RSSA 2 scheduling policy includes the biggest spread of mean throughput values between all priority classes, for the highest sub–carrier weight value the advanced RSSA 2 does not assign much more often such a sub–carrier to priority class one than to priority class two. When observing the total sub–carrier weight distribution of advanced RSSA 2 (Figure 4.6) it is seen, that the distributions of priority class one and three stay almost the same compared to the distribution plot of the simple RSSA (Figure 4.2), only the distribution of priority class two changes such that it has now almost the same distribution as the original one has of total sub–carrier weights. Since the choosing function for the sub–carriers is less strict here, priority class one gets almost the same overall distribution of sub–carrier states assigned as in the case of the simple RSSA. But still the choosing function is strict enough to increase the throughput of priority class three significantly. This increase though is a direct consequence of a decrease of priority class two. For the distribution, the maximum value is lower compared to the simple RSSA distribution and the graph is much wider, since it gets assigned now a much larger variety of sub–carrier weights.

When comparing the individual sub–carrier weight distributions of the advanced RSSA 3 (Figure 4.5) with the plot of the simple RSSA (Figure 4.1) we observe, that the advanced RSSA 2 version provides priority class one with the biggest share of sub–carriers owning the highest possible sub–carrier weight. Also, here priority class two gets a much lower ratio of such sub–carriers than in the case of the other advanced RSSA versions. When comparing the distributions for the total sub–carrier weights of the advanced RSSA 3 (Figure 4.8) with the distribution of the simple RSSA (Figure 4.2), it is interesting to observe again that the distribution plot of the advanced RSSA 3 scheduling policy is the most unsteady graph of all.

4.5 Upper Throughput Limits of Scheduling Policies for the M–ary Sub–Carrier State Model with a Gaussian SNR Distribution

This discussion is highly related to Section 3.5 and the corresponding section in [7]. The upper limits obtained here result from the same policies introduced in Section 3.5 as short–term and long–term fair optimal scheduling policies. With the settings of Section 4.3 for the M–ary sub–carrier state model with a Gaussian SNR distribution we obtain by simulation the results in Equation 4.4 and 4.5.

$$D_{\text{long–term fair optimal}} = 0.76 \cdot \frac{n \cdot b}{T_s} = 24.32 \frac{\text{bits}}{T_s} \tag{4.4}$$

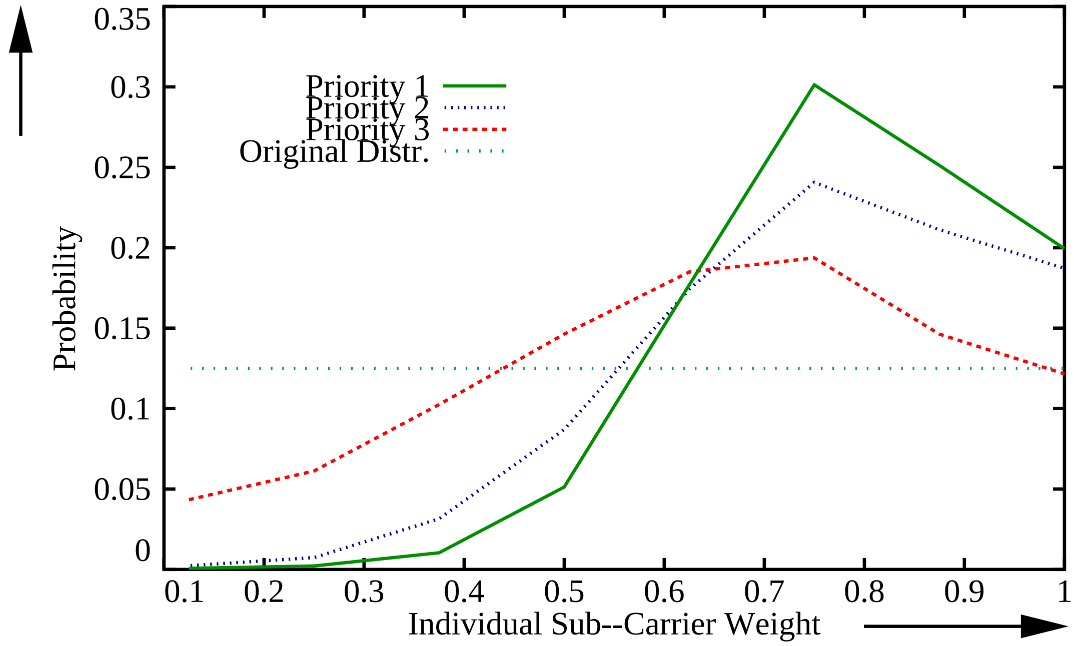

Figure 4.5: Distribution of Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 2 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

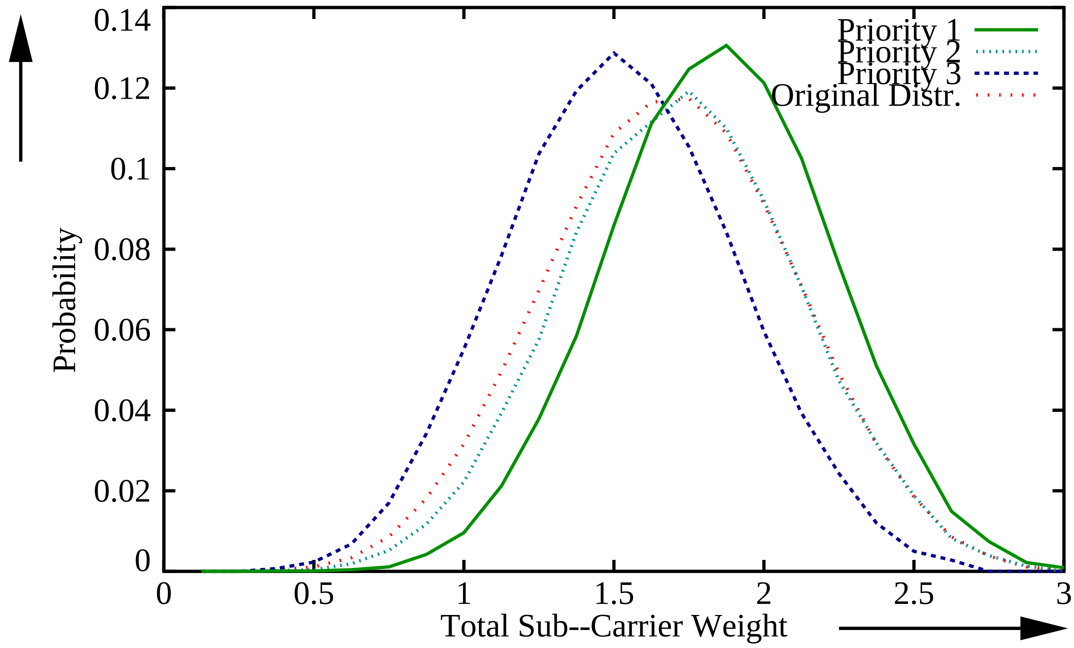

Figure 4.6: Distribution of the Total Sub–Carrier Weights per Priority Class for the advanced RSSA 2 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

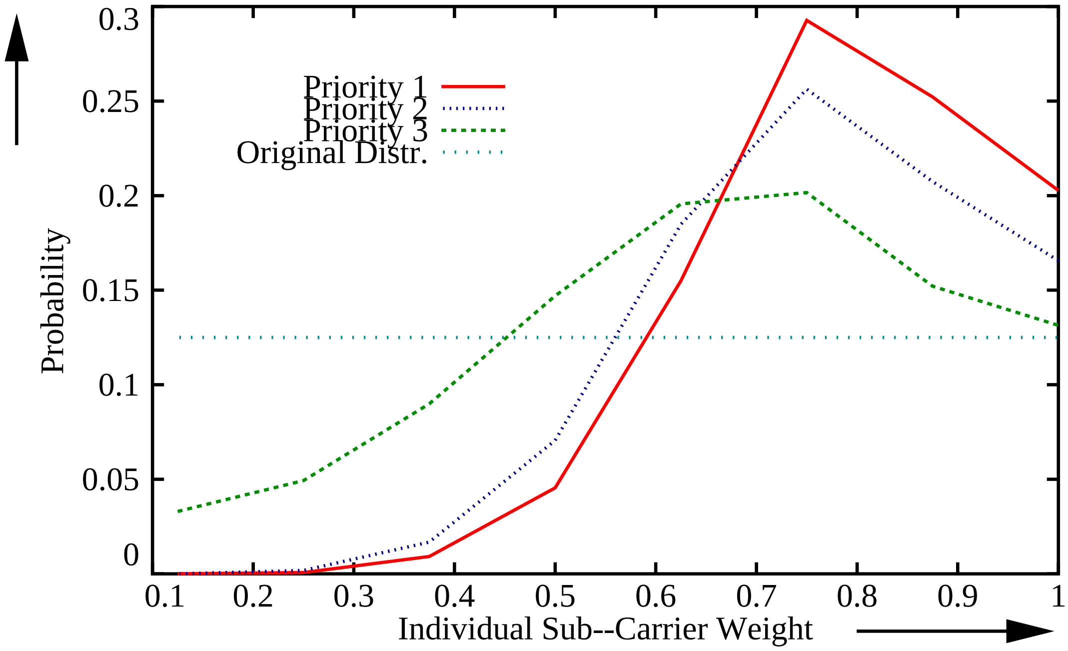

Figure 4.7: Distribution of the Individual Sub–Carrier Weights per Priority Class for the advanced RSSA 3 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

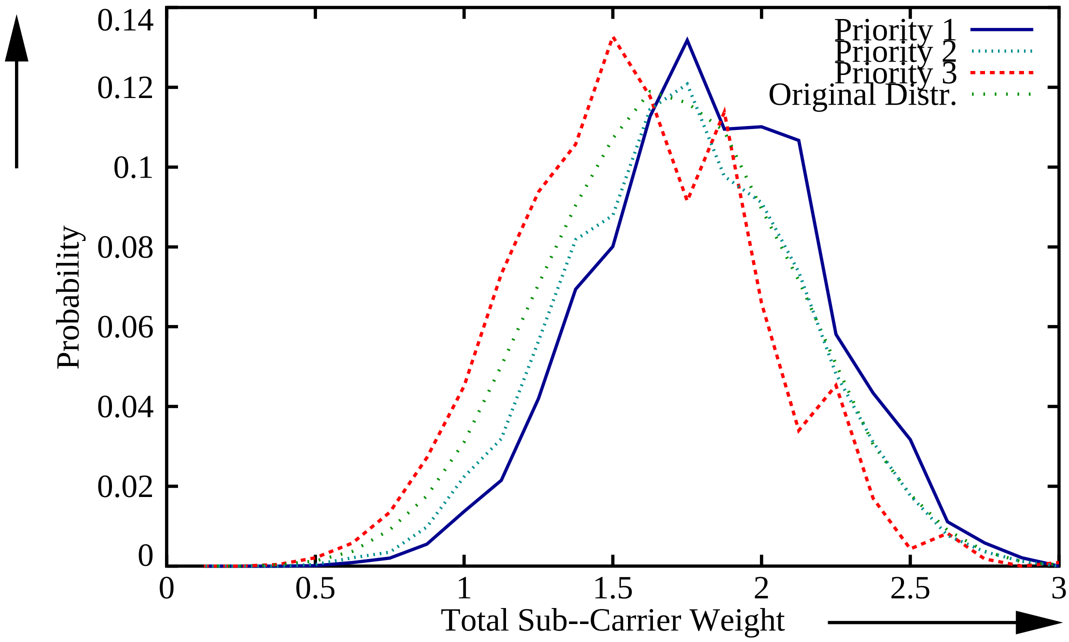

Figure 4.8: Distribution of the Total Sub–Carrier Weights per Priority Class for the advanced RSSA 3 with $J = 3$, $N = 4$, $S = 12$, and $M = 8$

Algorithm	Normalized Throughput (long–term fair optimal)
SSA	0.7401
Simple RSSA	0.922
Advanced RSSA 1	0.9193
Advanced RSSA 2	0.9418
Advanced RSSA 3	0.9318

Table 4.3: Normalized Throughput Results related to the long–term fair optimal scheduling policies for the M–ary sub–carrier state model with a Gaussian SNR distribution

Algorithm	Normalized Throughput (short–term fair optimal)
SSA	0.75
Simple RSSA	0.935
Advanced RSSA 1	0.9314
Advanced RSSA 2	0.9542
Advanced RSSA 3	0.9441

Table 4.4: Normalized Throughput Results related to the short–term fair optimal scheduling policies for the M–ary sub–carrier state model with a Gaussian SNR distribution

$$D_{\text{short–term fair optimal}} = 0.7501 \cdot \frac{n \cdot b}{T_s} = 24.01 \frac{\text{bits}}{T_s} \quad (4.5)$$

If we build now the ratio between the two optimal policies introduced here and the results obtained so far for the SSA, the simple RSSA and the advanced RSSA scheduling policies, we obtain the Tables 4.3 and 4.4 .

Compared to the values of the uniform SNR distribution in Table 3.3 and 3.4, the values achieved here are slightly better. Especially for the advanced RSSA 2 scheduling policy, the mean throughput is closer to the optimal throughput values. Beside this Table 4.3 and 4.4 show as in the uniform SNR distribution case that the applied scheduling policies work quite well, taking the fairness and complexity constraints into account.

Chapter 5

Conclusions

Several results can be drawn from the project outcomes so far.

- As in [7] dynamic scheduling policies can outperform the static scheduling policy equal to pure FDMA. For the simple RSSA throughput increase is around 31 % for the uniform distribution and around 25 % for the Gaussian distribution. This is much more than in the case of the binary sub–carrier state model, where the throughput increase was around 7 % compared to the throughput of pure FDMA. This suggests, that in case of a more realistic sub–carrier state model the simple RSSA performs even better.

- If we compare the throughput increase between the simple RSSA and the best advanced RSSA version (which is version 2) in the case of the M–ary model, throughput increase is around 2 % for both distributions used. For the binary sub–carrier state model, throughput increase caused by the advanced RSSA was around 3 % compared to the throughput of the simple RSSA. This corresponds to the already mentioned problem of advancing the RSSA in the case of the M–ary state model (refer to Section 3.4). In case of the M–ary model a further improvement in addition to the already well working simple RSSA is more difficult than in the case of the binary model.

- When comparing the efficiency of the advanced RSSA scheduling policy for the binary model with the efficiency achieved by the advanced RSSA 2 for both M–ary models, we observe that in the case of the binary model an efficiency of 99 % is achieved, while for the M–ary models an efficiency of 95 % is achieved. This is due to the more realistic but also more complex sub–carrier state model. For even more realistic models, including for example a correlated behavior of the sub–carrier states, we expect the efficiency to decrease even further, if no additional mechanisms are developed.

- When comparing the behavior for the different M–ary state models assumed here, we observe a higher throughput increase for the simple RSSA in case of the uniform distribution than in case of the Gaussian distribution when comparing to the static scheduling policy. This is interesting due to the fact that for the examples chosen in 3.3 and 4.3 the throughput values correspond to each other. However, the efficiency achieved by the static scheme is much higher in case of the Gaussian distribution. Fortunately efficiency achieved by the simple RSSA is in both SNR distribution cases quite similar as well as the efficiency achieved by the best advanced RSSA version. This points to quite a stable behavior even in different statistical situations.

- All throughput increases are result of the dynamic assignments of sub–carriers to wireless terminals. The down–side of the throughput increase is the need for signaling the new assigned sets to the wireless terminals. In contrast to the static scheduling policy, which only needs at some initial point to signal the set of fixed wireless terminals, the dynamic scheduling schemes have to signal prior to each transmission cycle the new assigned sets of sub–carriers. Therefore signaling overhead is significant.

- In order to use even more realistic sub–carrier state models, a model with a correlated sub–carrier state behavior should be introduced. However, the authors believe, that this will not effect the pure throughput values, as long as perfect sub–carrier state knowledge may be assumed.

- A clear trade–off between fairness, complexity and achieved throughput was found. The advanced RSSA is characterized by achieving almost 95 % of the theoretical upper throughput limit while still fulfilling a short–term fairness constraint and being quite low in terms of complexity. If complexity can be neglected while still assuming short–term fairness, throughput can be increased by 6 %. If fairness is neglected on a short time scale while still assuming a low complexity, throughput may be increased by 7 %. However, for all three combinations signaling stays the same.

- Therefore the goal of the authors is to investigate further mechanisms, which reduce signaling while still achieving solid throughput improvements and fulfilling the fairness and complexity constraints.

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