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Channel State Dependent Scheduling  
Policies for an OFDM Physical Layer  
using a Binary State Model

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## Abstract

In this report we investigate simple scheduling mechanisms to be used in wireless networks utilizing an **Orthogonal Frequency Division Multiplexing** (*OFDM*) physical layer. For the sub-carriers we assume a statistical model that consists of two possible states per sub-carrier towards each wireless terminal. As traffic model we assume pure constant bit rate streams. The streams are considered to be down-link oriented only and we have one stream for each wireless terminal. As scheduling policies we consider mainly a priority based dynamic assignment approach and compare the throughput results with a static assignment scheme, equal to pure **Frequency Division Multiple Access** (*FDMA*). Beside throughput, which represents the main metric of performance, constraints are also on signaling, fairness and complexity. Simulations and theoretical analysis show a significant performance increase for the dynamic scheduling policies compared to the static one. As signaling increases for the dynamic scheduling policies, the trade off between signaling overhead and throughput increase has to be taken into consideration.

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# Chapter 1

## Introduction

The demand for wireless communication systems has increased exponentially in the last few years [5]. While telephony was the major application for wireless networks of the first and second generation, third generation mobile networks will provide in addition data services with limited throughput. Intersymbol Interference (*ISI*) due to multi-path propagation is one of the major issues limiting the maximal throughput. In order to avoid this shortcoming, multi-carrier transmission schemes have been proposed within the last couple years as next generation physical layer transmission scheme. In particular **O**rtogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) has received special interest. By splitting a frequency band into a lot of sub-carriers and transmitting data through all these sub-carriers with data rates being much smaller compared to the overall data rate, the single symbol duration becomes large such that *ISI* is no longer a critical issue.

One future application of *OFDM* will be multi-user scenarios, for example cellular systems. For such scenarios a multiplexing scheme for the down-link and a **M**edium **A**ccess **C**ontrol (*MAC*) scheme for the up-link is necessary. In general one of the widely known schemes may be applied, which are **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*), **T**ime **D**ivision **M**ultiple **A**ccess (*TDMA*) and **C**ode **D**ivision **M**ultiple **A**ccess (*CDMA*). While scheduling schemes based on an *OFDM*-*TDMA* system already have been investigated [14], scheduling schemes based on an *OFDM*-*FDMA* system have not been described before. In an *OFDM*-*FDMA* system, the complete set of sub-carriers is divided into several subsets of sub-carriers. Afterwards these subsets of sub-carriers are assigned to wireless terminals.

In general a scheduling policy has to assign sub-carriers in an efficient way to wireless terminals in order to maximize or fulfill some performance metric. In order to do so, in this report we assume the *OFDM* physical layer to provide the scheduling unit with estimates of the sub-carrier states prior to each transmission. The main question throughout this technical report is, if dynamic scheduling policies in an *OFDM*-*FDMA* system can outperform static scheduling policies in such a system.

The remaining technical report is structured as the following. In Chapter 2 we give a brief introduction to *OFDM*. In Chapter 3 we then review some related work and discuss some properties of scheduling in wireless networks. After that, we introduce in Chapter 4 the investigated basic scenario and the statistical model of the sub-carrier behavior. In Chapter 5 we then present the investigated scheduling policies, show some analytical results of their throughput behavior, compare these results, and also discuss a upper throughput limit. In

Chapter 6 we finally discuss some conclusions and present some further work.

## Chapter 2

# Multi-Carrier Modulation Schemes and OFDM

As the need for higher and higher data rates in wireless networks becomes more obvious, single-carrier modulation schemes suffer more and more from Intersymbol Interference (*ISI*). If a high bandwidth is required in single-carrier modulation schemes, the symbol duration  $T_s$  of the used modulation will be considerably small. In general in wireless transmission schemes each received symbol will constitute of the original sent symbol, which might have suffered from attenuation, as well as of multiple delayed copies of this original sent symbol, which might also have suffered from attenuation. Also previous or later sent symbols might be part of the received symbol, if ISI occurs. The amount of copies that will interfere with the original sent symbol depends on the delay characteristics of the wireless channel. The delay characteristics of the channel is dependent on the amount and length of propagation paths between transmitter and receiver (multi-path propagation). Many paths result in many copies. A rather short path and a rather long path result in a relative big time spread between minimal channel delay  $\tau_{min}$  and maximal channel delay  $\tau_{max}$ . Denote the maximum delay spread by  $\tau_{max} - \tau_{min}$ . ISI occurs always if the maximum delay spread of the channel is bigger than the symbol duration:

$$\tau_{max} - \tau_{min} > T_s.$$

Until the maximum delay spread of the channel is a lot bigger than the symbol duration, ISI is not critical to the transmission since equalizers can compensate the occurring ISI [2]. However, equalization complexity increases as the ratio between maximal delay spread and symbol duration increases. Therefore, ISI becomes critical if the maximal delay spread is a lot bigger than the symbol duration, therefore if:

$$\tau_{max} - \tau_{min} \gg T_s.$$

As discussed above, in single-carrier modulation schemes with high data rates the symbol duration has to be rather small. Therefore the ratio between maximal delay spread and symbol duration is big and as a consequence ISI decreases the received symbol quality significantly.

As an example the maximum delay spread of a typical **D**igital **A**udio **B**roadcasting (*DAB*) wireless channel is  $\tau_{max} - \tau_{min} = 50\mu s$  whereas the symbol duration required for a single-carrier modulation system would be  $T_s = 0.5\mu s$  [3], [4].

In contrast to this situation of high speed broadband single-carrier modulation schemes, low speed narrow-band single-carrier modulation schemes do not suffer from ISI. This is mainly due to the fact that the symbol duration  $T_s$  might be still smaller than the maximum delay spread of the wireless channel, but the the factor between the two values is not that big any more:

$$\tau_{max} - \tau_{min} \geq T_s.$$

In such a case, equalization complexity is low and the occurring ISI can be completely removed from the received signal.

For example in the **G**lobal **S**ystem for **M**obile **C**ommunications (*GSM*) a maximum delay spread of  $\tau_{max} - \tau_{min} = 20\mu s$  was assumed when designing the system originally. The symbol duration of the used **G**aussian **M**inimum **S**hift **K**eying (*GMSK*) modulation is  $T_s = 4\mu s$ . While the maximum delay spread is still bigger than the symbol duration, the complexity of an efficient equalizer for such a scenario with a ratio of 4 is much smaller than for a scenario with a ratio of 100 as in the DAB case. Therefore ISI is no problem in GSM.

From this the principle of a multi-carrier modulation scheme is quite obvious. A broadband carrier is divided into many narrow-band carriers. Accordingly the high rate data stream is divided into several parallel low rate data streams. The lower data rate streams modulate the narrow-band carriers. If a broadband carrier is divided into  $N$  narrow-band carriers, the data rate per narrow-band carrier is reduces by the factor of  $N$ . This way the symbol duration for each narrow-band carrier is increased by the factor of  $N$  and subsequently the impact of ISI is reduced. We will refer to these narrow-band carriers as sub-carriers.

OFDM is characterized by splitting a broadband carrier into sub-carriers, which are all mutually orthogonal. In order to achieve the orthogonality, the frequencies of the sub-carriers have to fulfill a certain distance constraint towards each other, depending on the symbol length intended to convey through each sub-carrier. If the symbol length for each of the sub-carriers is  $T_s$ , then the frequency distance  $\Delta f$  between two adjacent sub-carriers has to be:

$$\Delta f = \frac{1}{T_s}.$$

Here we assume that the symbol rate is equal to the baud rate of the transmission system.

## Chapter 3

# Scheduling Policies for Wireless Networks

Sharing resources between different users is one of the foundations of computer networks. However, sharing automatically introduces the problem of contention for the shared resource as a result of statistical fluctuations of resource requests. A scheduling policy is used in order to decide which request to assign the shared resource next out of a set of active resource requests, which are stored in a service queue.

For cellular systems, which are the application scenario considered in this report, the shared resource is the wireless link between the access point and the wireless terminals in the cell. As managing unit the access point is responsible for scheduling. Since this report investigates OFDM-FDMA systems, the shared resource is the set of sub-carriers in the OFDM system. The scheduling policy has to answer the question how to build the subsets of sub-carriers out of the total set of sub-carriers and how to assign these sets for each transmission cycle.

The question how to assign the shared resource, i.e. the wireless link, in a wireless system has been widely discussed in the last several years. For wired systems the question is also interesting, but is easier to answer due to the reliability of wired links. In contrast, wireless links are much more unreliable and can cause severe transmission errors while a transmission at some other time does not cause errors at all. The reasons for this unreliability are mainly :

- A location-dependent varying signal strength. This is caused by destructive interference of multiple copies of the same sent signal, which are propagated on multiple paths from transmitter to receiver, which relies more or less on the reflection of radio waves from buildings and objects in the area of transmission.
- A time-dependent varying signal strength. This is caused by the possible mobility of objects (including the transmitter and/or receiver) within the transmission area.

Therefore the unreliability of the wireless link is an additional constraint, which has to be taken into account when discussing scheduling policies for wireless networks. As a consequence, the idea of channel-state dependent scheduling policies has been introduced.



It suggests to include channel-state information in the scheduling decision, which wireless terminal to assign the shared resource next. Therefore the scheduler is provided either with channel-state information obtained by prediction units such as channel estimators or by probing the wireless channel prior to each transmission cycle by a RTS/CTS handshake for example.

An early work on channel-state dependent scheduling in connection with improving TCP performance was done by Bhagwat et al. in [10]. Here the problem of head-of-line blocking due to a bad channel state is investigated. Due to a bad wireless channel state, it may happen that a packet has to be transmitted several times before it is acknowledged. If in such a system a **F**irst-**C**ome **F**irst-**S**erve (*FCFS*) scheduling policy is active, these multiple transmission attempts will cause a blocking at the head of the output queue. Due to the location-dependent error nature of wireless channel, the recipients of other packets in the output queue behind the blocking packet could be in a good wireless channel state and therefore could have received their packets while the system tries to transmit the packet in the first queue position. Therefore both, average latency and channel utilization, can be improved if such packets with strong link quality to recipients are given priority instead of following a strict FCFS policy. Several scheduling policies were investigated and performance evaluated via discrete event simulation. The found improvements demonstrate the advantage of channel-state dependent scheduling.

An early experimental work was done by Desilva et al. in [9]. Here a wireless LAN is modified such that channel-state dependent scheduling can be compared with the original transmission policy. In short, the used wireless LAN employs a **C**arrier **S**ense **M**ultiple **A**ccess with **C**ollision **A**voidance (*CSMA/CA*) scheme in order to assign the shared wireless link to stations who have data in a queue to be transmitted. In addition to this each station is now able to sense the state of the wireless channel to some neighbor of it. If the station has some data to be transmitted to some other station and the wireless channel towards this station is actually bad, transmission is delayed and the station will not try to obtain a slot in the CSMA/CA scheme. While the station delays the data, it continuously senses the channel to the receiving station until the channel recovers. Then the transmission is done. While data is delayed in a special marked queue, data destined for an other station actually being in a good channel state may be transmitted from the same station. Experimental performance evaluation with UDP streams and FTP session demonstrate significant performance benefits of channel-state dependent scheduling.

Due to the varying signal strength of wireless channels another objective of research was to develop models and algorithmic approximations which were able to provide fairness within wireless networks. A well known model called **w**ireless **F**luid **F**air **Q**ueuing (*wFFQ*) was introduced by Bharghavan et al. in [7]. They also introduce an algorithmic approximation called **W**ireless **P**acket **S**cheduling (*WPS*) which includes the idea of channel-state dependent scheduling. A key difference of the new model and its approximation is the idea of compensating back-lagging data flows which suffered from bad channel states by reducing capacity of leading data flows, which gained temporarily more capacity than their fair share would allow in a strict scheduling system. These assigned capacity differences are caused by a channel-state dependent scheduling policy. Compensation mechanisms also rely on channel-state dependent scheduling. Similar work was done by Stoica et al. in [8].

In wireless networks the concept of channel-state dependent scheduling policies shows

a lot of advantages compared to policies not including channel-state information in their scheduling decisions. However, channel-state dependent policies introduce some signaling overhead to the system in order to avoid transmissions while the channel state is bad. In order to build and assign the subsets of the OFDM sub-carriers in the OFDM-FDMA system, we will apply channel-state dependent policies and investigate their behavior in terms of some performance metrics.

## Chapter 4

# Scenario Description and Assumptions

Let us assume a wireless cellular system. The physical layer is using **Orthogonal Frequency Division Multiplexing (OFDM)** as transmission scheme. For each cell an access point organizes all downstream and upstream data transfers regarding the wireless terminals located in that cell. In general we assume a total amount of  $J$  wireless terminals to be located within the cell. For illustration purposes we often will set  $J = 3$ . Furthermore we will exclusively investigate cases where the access point is handling downstream data traffic. In this report we focus on down-link transmissions to achieve upper-bound results initially. In particular we assume that each wireless terminal has exactly one data transfer session active. All  $J$  data streams are equal in their quality of service requirements. The bit rate is considered to be constant.

For the OFDM physical layer we will assume the following. Our cell provides  $S$  sub-carriers in total which can be assigned arbitrarily towards  $J$  wireless terminals. Each sub-carrier is described through a state towards each wireless terminal in the cell, where the specific state towards a wireless terminal is in general dependent on the **Signal-to-Noise Ratio (SNR)** measured for this sub-carrier towards the wireless terminal. Each sub-carrier is described through  $J$  SNR states, one state for each wireless terminal.

For the sub-carrier state behavior we consider the following statistical model: If the SNR of a sub-carrier is above a certain threshold for some wireless terminal, we consider this sub-carrier to be in a *good* state  $G$  regarding this wireless terminal. If the SNR of this sub-carrier is below that threshold, the sub-carrier is in a *bad* state  $B$  regarding this wireless terminal. For each wireless terminal each sub-carrier can be either in state  $G$  or in state  $B$ . Due to this binary character, the combination of states a sub-carrier is in towards each of the  $J$  wireless terminals may take one out of  $2^J$  possible combinations. In the case of  $J = 3$  eight different state combinations are possible for each sub-carrier as given in Figure 4.1. State behavior of different sub-carriers is assumed to be completely independent of each other. Also the state behavior between different wireless terminals within the same sub-carrier is assumed to be independent of each other. Sub-carrier states are considered to be constant during one symbol time interval. At any time the probability that a sub-carrier is in state  $G$  towards a wireless terminal is denoted by  $p_g$ . Therefore the probability that this sub-carrier is in state

$B$  towards the wireless terminal is  $1 - p_g$ . If a sub-carrier is in state  $G$  towards a particular wireless terminal, the access point may convey information to this wireless terminal through this wireless terminal. In order to do so, one fixed modulation type is available, where a symbol of the modulation alphabet represents  $b$  bits and has the a time duration (symbol time) of  $T_s$  (already including the guard interval of the OFDM modulation system). Therefore if a sub-carrier is in state  $G$  towards some wireless terminal, the access point may convey  $b$  bits to this wireless terminal on this sub-carrier. In state  $B$  transmission is not possible.

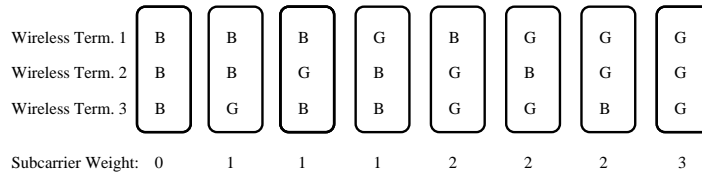


Figure 4.1: All State Possibilities per Sub-Carrier with their Weight for the binary Sub-Carrier State Model with  $S = 12$ ,  $J = 3$ , and  $N = 4$

Let us refer to this statistical model as the binary sub-carrier state model.

For the further discussion let us introduce here the notion of a sub-carrier weight. The sub-carrier weight describes the total amount of states  $G$  a sub-carrier owns at the transmission symbol time. For  $J$  wireless terminals a sub-carrier may have the weight 0, if the sub-carrier is in state  $B$  towards all three wireless terminals. At best, a sub-carrier has the weight  $J$ , if the sub-carrier is in state  $G$  towards all three wireless terminals. Figure 4.2 shows a possible setting within a cell for a fixed transmission symbol period.

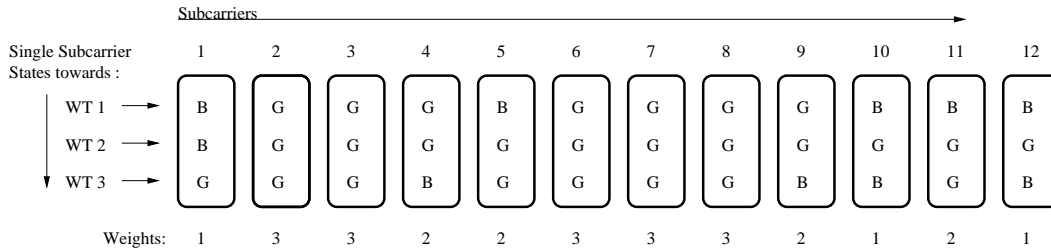


Figure 4.2: Example Sub-Carrier State Setting within one Cell for  $S = 12$ ,  $J = 3$ , and  $N = 4$

Some further general assumptions may be stated for the sub-carriers. Time is slotted into intervals of the duration  $T_s$ . Assignments between sub-carriers and wireless terminals may be changed between two consecutive time units. Prior to each time unit, the physical layer will provide estimates of the SNR for each sub-carrier for this time unit. For this scenario let us assume, that the estimates never differ from the real state of a sub-carrier towards some wireless terminal encountered afterwards during the time unit. Therefore perfect knowledge of the sub-carrier states is assumed prior to each time unit.

We will distinguish between two types of scheduling policies: static and dynamic ones. While the dynamic policies will include the perfect knowledge in their algorithmic behavior, the static policy will always neglect the usage of this knowledge. In Section 5.8 we will

also assume the case of unreliable estimates (i.e. non-perfect knowledge). This is marked explicitly. Scheduling will take place on a per-symbol time base, therefore utilizing the flexibility provided by the physical layer. As a second criterion for the investigated scheduling policies we will only consider policies guaranteeing fairness. In our case, since the quality of service requirements by the data streams are similar among the different wireless terminals, this only calls for the same average throughput per wireless terminal. Fairness will be provided by assigning the same amount of sub-carriers to each of the  $J$  wireless terminals for each time unit. As a fair share denote  $N$  as the amount of sub-carriers which should be assigned to each of the  $J$  wireless terminals. As a third criteria we will often mention the complexity of a scheduling policy, which should be as low as possible.

For the dynamic scheduling policies we further assume, that the signaling for the new sub-carrier assignments of each time unit does not influence the flexibility of the system. Obviously the signaling effort which would be required for scheduling on a per-symbol base would be very large if not impossible to realize. Due to the interest of investigating a best-case scenario we assume this effort to be negligible.

Scheduling policies will always try to maximize the throughput for all wireless terminals. This will serve as our primary performance metric. Beside the raw throughput expressions and values provided here, we also obtain an efficiency measure. We will refer to this efficiency ratio as normalized throughput. The detailed derivation of the normalized throughput is given later on in the paper.

## Chapter 5

# Scheduling Policies for the binary Sub-Carrier State Model

In this section we discuss the working principles of the static and the dynamic scheduling policies. Further we present some numerical results. Also an upper throughput limit is presented for the assumed statistical model of the sub-carriers. At the end the behavior of the policies is investigated for a varying probability of  $p_g$  and for non-perfect estimates.

### 5.1 Static Sub-Carrier Assignments – SSA

As a quite simple scheduling policy one can imagine a scheme where sub-carriers are assigned statically to wireless terminals. Such a scheme corresponds to pure FDMA. As mentioned in Section 4, in order to provide fairness, each wireless terminal will be assigned  $N$  sub-carriers in his set from the access point. This assignment will not be changed. Figure 5.1 illustrates the static sub-carrier assignment.

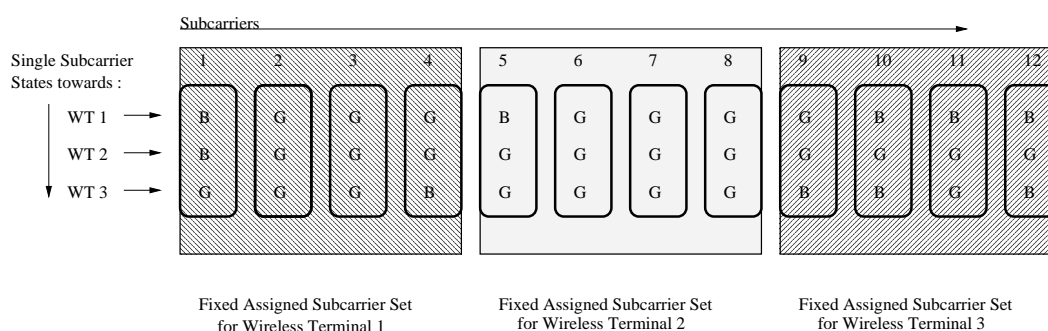


Figure 5.1: Example Sub-Carrier Assignment for the SSA Scheduling Policy with  $S = 12$ ,  $J = 3$ , and  $N = 4$

In order to obtain the mean throughput of this scheduling policy, let us first consider the following. Consider a statistical experiment which is repeated  $I$  times. Each experiment is independent of each other. Further the outcome of each experiment may be the event  $A$  or

not. No other outcome is possible. The probability for this outcome is denoted by  $p_a$ . Denote by  $X^n$  the amount of outcomes  $A$  in the  $I$  repetitions of the experiment. Then we know for the probability that  $X^n = k$  [13] :

$$P(X^n = k) = \binom{I}{k} \cdot (p_a)^k \cdot (1 - p_a)^{I-k} \quad (5.1)$$

This directly corresponds to our problem. Out of  $N$  sub-carriers (corresponding to the  $I$  repetitions) we desire the probability that  $i$  of them are in a state  $G$  (corresponding to  $k$  experiments which have the outcome  $A$ ). The probability that a sub-carrier will be in a good state is denoted by  $p_g$  and is equivalent for all sub-carriers (corresponding to the probability  $p_a$ ). Denote the number of good sub-carriers within the  $N$  total sub-carriers as  $X$  (corresponding to  $X^n$ ). Therefore we obtain for the probability that  $X = i$  during one time unit corresponding to 5.1 :

$$P(X = i) = \binom{N}{i} \cdot (p_g)^i \cdot (1 - p_g)^{N-i} \quad (5.2)$$

For the mean throughput we desire now the mean value of good sub-carriers out of the  $N$  total sub-carriers. From [13] we know for Equation 5.1 :

$$E(X^n) = \sum_{k=0}^I \binom{I}{k} \cdot k \cdot (p_a)^k \cdot (1 - p_a)^{I-k} = p_a \cdot I \quad (5.3)$$

This applies to Equation 5.2 as the following :

$$E(X) = \sum_{i=0}^N \binom{N}{i} \cdot i \cdot (p_g)^i \cdot (1 - p_g)^{N-i} = p_g \cdot N \quad (5.4)$$

With 5.4 we obtain for the mean throughput:

$$D_{\text{binary SSA}} = \sum_{i=0}^N \binom{N}{i} \cdot i \cdot (p_g)^i \cdot (1 - p_g)^{N-i} \cdot \frac{b}{T_s} = N \cdot p_g \cdot \frac{b}{T_s} \quad (5.5)$$

As we can see, the throughput is linearly dependent on the probability that a sub-carrier is in state  $G$ ,  $p_g$ . Also, the absolute throughput is linearly dependent on  $N$ , the amount of sub-carriers being available per wireless terminal.

## 5.2 Pseudo Static Algorithm – PSA

Intuitively, it seems in the above mentioned policy as if the throughput can be increased. If the access point could switch sub-carriers in their assignment sets in the case that one of the sub-carriers is in state  $B$  towards his assigned wireless terminal, but in a good state towards another wireless terminal then the overall throughput could be increased. The only problem with such a policy could be the fairness of switching sub-carriers between different wireless terminals. As Pseudo Static Algorithm we present here a policy that provides a fair switching scheme. As before in the SSA, each wireless terminal is initially assigned a static

sub-carrier set of  $N$  sub-carriers. But in the case that one of the assigned sub-carriers is in state  $B$  towards his wireless terminal, the access point will try to exchange this sub-carrier for a better one. As a switching rule, the access point is only allowed to change the bad sub-carrier with a sub-carrier being in state  $G$  regarding the first mentioned wireless terminal but in state  $B$  regarding the other two wireless terminals (i.e. the access point is only allowed to take a sub-carrier of weight 1). Therefore no other wireless terminal will suffer from the switching rule. If the access point can not find such a sub-carrier with weight 1 being only good for the initially mentioned wireless terminal, no sub-carrier switching will take place and less data will be conveyed due to a state  $B$  sub-carrier.

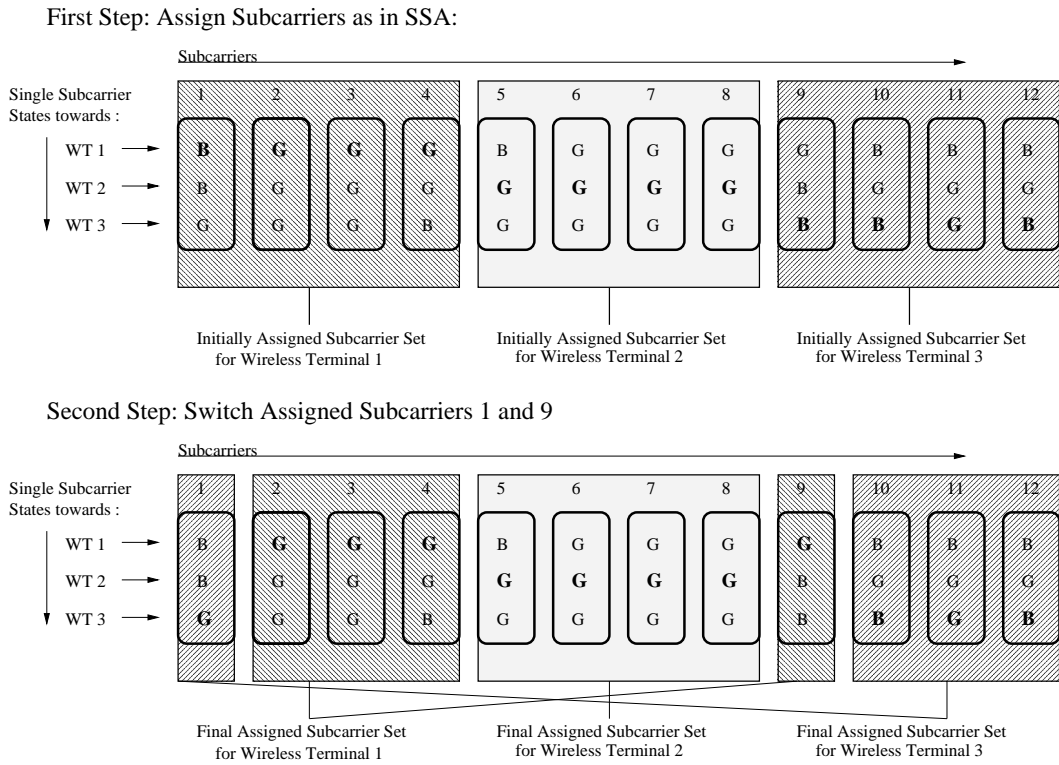


Figure 5.2: Example Sub-Carrier Assignment for the PSA Scheduling Policy with  $S = 12$ ,  $J = 3$ , and  $N = 4$

In order to derive the mean throughput of such a scheduling policy, let us consider in more depth the different combinations of available sub-carriers in state  $G$  and their probabilities of occurrence regarding some wireless terminal:

- All assigned  $N$  sub-carriers are in a good state towards this wireless terminal

$$P(\text{all sub-carriers in state G}) = (p_g)^N$$

Availabe good Sub-Carriers:  $N$

here: no need for a trade !



- All but one assigned sub-carrier ( $N - 1$  sub-carriers) are in a good state towards this wireless terminal

$$\begin{aligned}
 P(\text{one sub-carrier in a bad state, } N - 1 \text{ in a good state}) &= N \cdot (1 - p_g) \cdot (p_g)^{N-1} \\
 \text{Available good Sub-Carriers:} & \quad N - 1 \\
 P(\text{another sub-carrier is available out of the } S - N \text{ sub-carriers}) &= \\
 (S - N) \cdot p_g \cdot (1 - p_g)^{J-1} \cdot \left(1 - p_g \cdot (1 - p_g)^{J-1}\right)^{S-N-1} \\
 \text{Available sub-carriers in this special case:} & \quad N
 \end{aligned}$$

- Two sub-carriers are in a bad state, all other assigned ones are in a good state

$$\begin{aligned}
 P(\text{two sub-carriers in a bad state, all others good}) &= \binom{N}{2} \cdot (1 - p_g)^2 \cdot (p_g)^{N-2} \\
 \text{Available Sub-Carriers:} & \quad N - 2 \\
 P(\text{another sub-carrier is available out of the } S - N \text{ sub-carriers}) &= \\
 (S - N) \cdot p_g \cdot (1 - p_g)^{J-1} \cdot \left(1 - p_g \cdot (1 - p_g)^{J-1}\right)^{S-N-1} \\
 \text{Available sub-carriers in this special case:} & \quad N - 1 \\
 P(\text{two other sub-carriers are available out of the } S - N \text{ sub-carriers}) &= \\
 \binom{S-N}{2} \cdot \left(p_g \cdot (1 - p_g)^{J-1}\right)^2 \cdot \left(1 - p_g \cdot (1 - p_g)^{J-1}\right)^{S-N-2} \\
 \text{Available sub-carriers in this special case:} & \quad N
 \end{aligned}$$

- and so on

This scheme can be continued till all primarily assigned sub-carriers of the designated wireless terminal are in a bad state. In such a case the access point has to try to find up to  $N$  other sub-carriers, that can not be used by both other wireless terminals. From the combinations shown above, the mean throughput for the PSA ends up with:

$$\begin{aligned}
 D_{\text{binary PSA}} &= \sum_{i=0}^N \binom{N}{i} \cdot (1 - p_g)^{N-i} \cdot (p_g)^i \\
 &\cdot \left( i + \sum_{j=1}^{N-i} \binom{S-N}{j} \cdot \left(p_g \cdot (1 - p_g)^{J-1}\right)^j \cdot \left(1 - p_g \cdot (1 - p_g)^{J-1}\right)^{S-N-j} \cdot j \right) \cdot \frac{b}{T_s} \quad (5.6)
 \end{aligned}$$

When comparing Equation 5.6 with Equation 5.5, therefore comparing the mean throughput of the PSA with the mean throughput of the SSA, we see that the throughput term of the PSA is a sum of the throughput result for the SSA and an additional term. The additional term represents the case that a bad sub-carrier might be switched according to the defined rule. For a bigger amount of sub-carriers total where the amount of wireless terminals stays the same (a bigger  $S$  and therefore a bigger  $N$ ), the probability that a bad sub-carrier might be switched, increases. Therefore for an increasing  $S$  the throughput of the PSA increases of course as the throughput of the SSA does, but it increases even stronger, since the switching probability increases.

The mentioned switching rule for sub-carriers is quite simple. It is not very difficult to think of more complex but also more efficient switching rules. The access point could for

example try to switch sub-carriers that do not necessarily have the weight one. Instead, the access point could try to switch maybe a couple of sub-carriers such that at the end every wireless terminal can receive a better throughput. However, for such schemes the provision of fairness increases the complexity. In the opinion of the authors, the complexity introduced by such fair switching rules does not pay off in terms of throughput. As already observed in Equation 5.6, the analytical derivation is rather difficult. More complex switching rules would include even longer and more complicated terms, which gives a hint to the complexity of the scheme. Due to this, other switching rules were not considered for the PSA scheduling policy.

### 5.3 Simple Rotating Sub-Carrier Space Algorithm – Simple RSSA

As mentioned before the main weakness of the PSA policy is the increasing complexity in order to switch sub-carriers efficiently between assignment sets of different wireless terminals while still providing fairness. In order to solve this problem, a different approach is suggested. As another dynamic policy we can think of the following. The access point could choose  $N$  sub-carriers being in state  $G$  towards some prioritized wireless terminal out of  $S$  sub-carriers total. By this we would introduce priorities, where this wireless terminal would have the highest priority. According to this, for the wireless terminal with the second highest priority the access point would then have to choose out of  $S - N$  remaining sub-carriers the ones being in a state  $G$  towards that wireless terminal. For the wireless terminal with the lowest priority class the access point would assign the remaining  $N$  sub-carriers. If for any priority class the access point can not find  $N$  sub-carriers being in state  $G$  towards that priority class wireless terminal, the access point will assign also some sub-carriers being in state  $B$  towards this wireless terminal. Therefore, each priority class wireless terminal always receives  $N$  assigned sub-carriers.

Obviously this policy would prefer the wireless terminal of priority class one. But by giving this priority periodically to every wireless terminal (every  $J$ -th symbol time interval each wireless terminal would be the one with the highest priority), this policy would become fair.

For the derivation of the mean throughput, we only have to obtain the single mean throughput values for the different priority classes and then have to build the mean for obtaining the overall throughput. The calculations are quite simple since we basically encounter the binomial combination problem. For illustration purposes we will set  $J = 3$ . Note for the derivations shown below, that for priority class one and two we just calculate the probability, that at least  $N$  sub-carriers are in state  $G$ . Therefore we only include the case, that class one and two will be assigned  $N$  good sub-carriers, never less than  $N$  good ones. Since there will be cases, where the upper two priority classes will only receive less than  $N$  good sub-carriers, the shown analysis provides us with a lower throughput limit. The reason for doing so is first of all analytical simplicity. Second, as seen in the next Section, depending on the values of  $N$  and especially  $p_g$ , the throughput results are almost not effected. For a value of  $p_g = 0.9$ , which will be assumed in the next section, throughput results for the upper two priority classes are already very good, such that the lower limit shown here is sufficient for

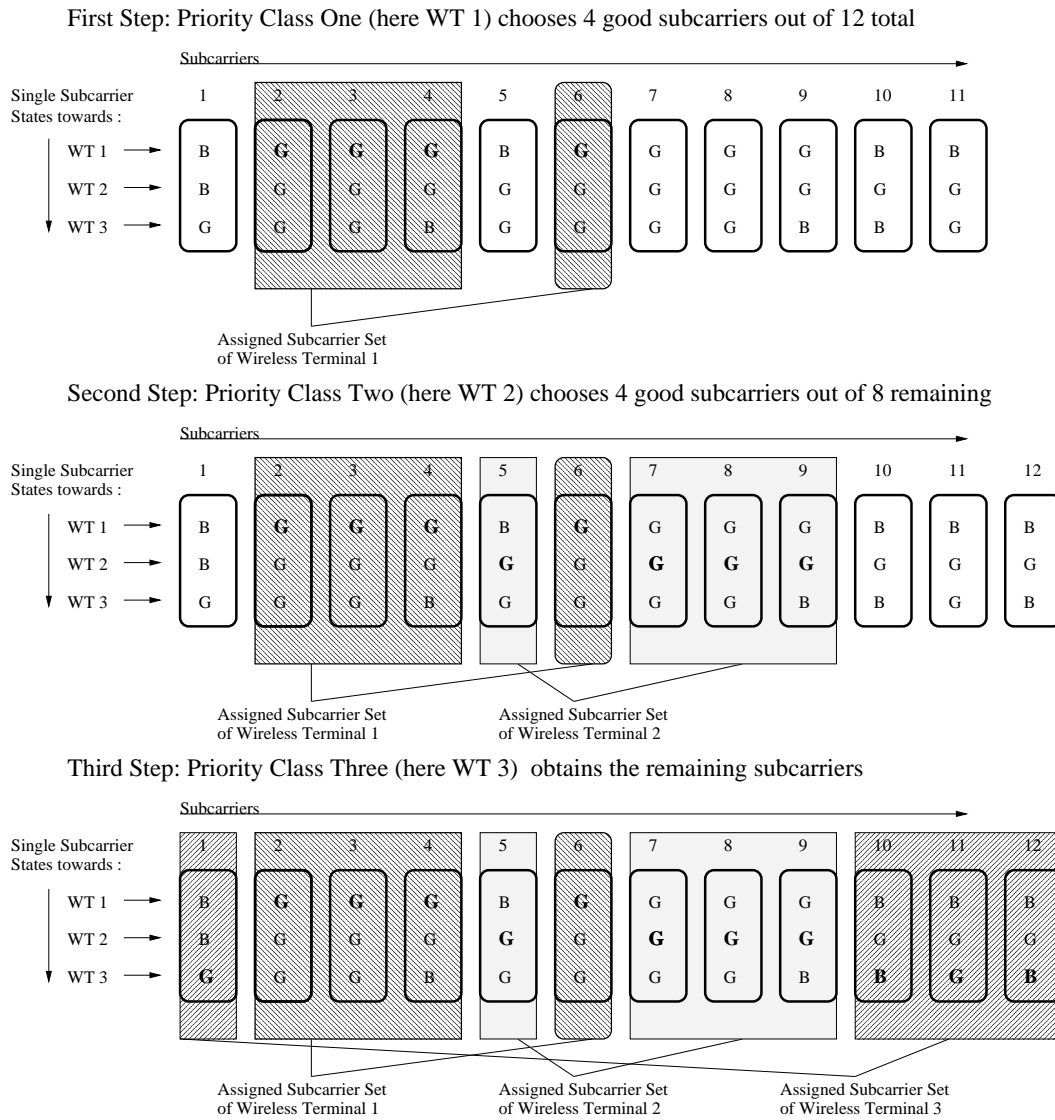


Figure 5.3: Example Sub-Carrier Assignment for the simple RSSA Scheduling Policy with  $S = 12$ ,  $J = 3$ , and  $N = 4$

the further discussion. However, for smaller values of  $p_g$  the lower limit does not correspond that strong with simulated throughput values any more (5.4).

Technically the derivations of the throughput is done by obtaining the probability of less than  $N$  sub-carriers being in state  $G$  and then subtracting this result from 1. This results in the probability, that priority class one and two will receive  $N$  good sub-carriers. We obtain:

- Throughput of the wireless terminal with the first priority (available sub-carrier space is  $S = J \cdot N = 3 \cdot N$ ):

$$D_{first} = \left( 1 - \sum_{i=0}^{N-1} \binom{3N}{i} \cdot p_g^i \cdot (1 - p_g)^{3N-i} \right) \cdot N \cdot \frac{b}{T_s}$$

- Throughput of the wireless terminal with the second highest priority (available sub-carrier space is  $S - N = (J - 1) \cdot N = 2 \cdot N$ ):

$$D_{second} = \left( 1 - \sum_{i=0}^{N-1} \binom{2N}{i} \cdot p_g^i \cdot (1 - p_g)^{2N-i} \right) \cdot N \cdot \frac{b}{T_s}$$

- Throughput of the wireless terminal with the third priority:

$$D_{third} = \sum_{i=1}^N \left( \binom{N}{i} \cdot (1 - p_g)^{N-i} \cdot (p_g)^i \cdot i \right) \cdot \frac{b}{T_s}$$

For the overall throughput we obtain the mean of the three priority classes above:

$$D_{\text{binary SRSSA}} = \frac{D_{first} + D_{second} + D_{third}}{3} \quad (5.7)$$

As stated above, due to illustration purposes we only obtained the results for  $J = 3$ . Now let us briefly discuss the impact of more wireless terminals in the cell. If we consider a case of some  $J$  wireless terminals in the cell, where  $J$  is bigger three and the basic condition  $S = J \cdot N$  still holds, the mean throughput results can only increase per wireless terminal. This is due to the following reason. Since  $S = J \cdot N$  is assumed, more than 3 wireless terminals in the cell automatically means that more overall sub-carriers ( $S$ ) are available. More overall sub-carriers leads to the situation, that the access point can pick now good sub-carriers out of more overall sub-carriers towards the wireless terminals being actually in the upper priority classes. Therefore the probability, that such a wireless terminal will receive  $N$  good sub-carriers increases and such does the mean throughput too. The throughput of the three lowest priority classes will remain as stated above for the special case of  $J = 3$ . By this the overall mean throughput, which is just the mean of the priority class mean throughputs, will increase for the case of  $J > 3$ .

Comparing the mean throughput of the simple RSSA with the SSA, we can see first of all that the last priority class has the same throughput as the SSA does. But the throughput of the simple RSSA consists of the throughput values of the upper priority classes, too. Because the upper priority classes have a bigger sub-carrier space to get good sub-carriers assigned from, the overall throughput of the simple RSSA will be bigger than in the case of the SSA.

For an increasing number of sub-carriers  $S$  with a constant number of wireless terminals, the throughput of the simple RSSA will increase. It will increase stronger than the throughput of the SSA, since the sub-carrier space for the upper priority classes increases, which leads to a higher probability that an upper priority class will be assigned  $N$  good sub-carriers.

## 5.4 Example Setting and first Comparison of the Policies

Another way to compare the dynamic policies versus the static policy is to discuss an example setting. Assume for the cell  $J = 3$  and  $N = 4$ , therefore  $S = 12$ . For the symbol length of the used modulation scheme we will consider  $b = 8$  bit, as in 256 QAM for example. Furthermore, for the sub-carrier state probability we use  $p_g = 0.9$ . Just for reasons of judging the results, we can calculate the throughput in the case that all sub-carriers are always in state  $G$ . Obviously this will end up in a throughput per wireless terminal of:

$$D_{\text{perfect Sub-Carriers}} = \frac{32}{T_s} \text{ bits}$$

For the SSA policy, the throughput per wireless terminal will be:

$$D_{\text{binary SSA}} = 3.6 \cdot \frac{b}{T_s} \text{ bits} = \frac{28.8}{T_s} \text{ bits}$$

For the PSA, we obtain:

$$D_{\text{binary PSA}} = 3.6236 \cdot \frac{b}{T_s} = \frac{28.988}{T_s} \text{ bits}$$

For the simple RSSA , we obtain:

$$\begin{aligned} D_{\text{first}} &= 4 \cdot \frac{b}{T_s} & D_{\text{second}} &= 3.9983 \cdot \frac{b}{T_s} & D_{\text{third}} &= 3.6 \cdot \frac{b}{T_s} \\ \Rightarrow D_{\text{binary SRSSA(Ana.)}} &= 3.8661 \cdot \frac{b}{T_s} = \frac{30.9288}{T_s} \text{ bits} \end{aligned}$$

These result were all obtained by the shown analysis. In order to verify the analysis, a simulation framework was programmed where all scheduling policies were implemented in. The simulation results match with the analytical results within the confidence interval. Therefore these analytical results are backed through simulations processed for all approaches.

For the simple RSSA, the analytical expressions are lower limit values. In order to show the quality of the lower limit, we present here the results for the the simulation of this scheduling policy. We obtained:

$$\Rightarrow D_{\text{binary SRSSA(Sim.)}} = 3.8663 \cdot \frac{b}{T_s} = \frac{30.9304}{T_s} \text{ bits}$$

As expected the value for the simulation is bigger than the value of the analytical result (see also Section 5.7).

As we can see, the simple RSSA can utilize the total offered bandwidth more efficient, therefore providing each wireless terminal a better throughput. We can express the throughput results of the different policies in percentages of the throughput of the perfect sub-carrier

Algorithm	Ratio between Throughput and Throughput in the perfect sub-carrier case
SSA	0.9
PSA	0.905
Simple RSSA	0.966

Table 5.1: Throughput Results for the first Scheduling Policies of the binary Sub-Carrier State Model for  $N = 4$ ,  $p_g = 0.9$ , and  $J = 3$

case. Although this does not represent the upper throughput limit (refer to Section 5.6), the ratio that we obtain can help us in judging the results obtained so far. Table 5.1 holds the values.

For the simple RSSA we can observe, that the mean throughput values are already quite high for the upper two priority classes. Therefore the lower limit (as discussed in Section 5.3) achieves already very good results for the case of  $p_g = 0.9$ .

## 5.5 Advanced Rotating Sub-Carrier Space Algorithm – Advanced RSSA

As we have seen, the simple RSSA can achieve quite a good throughput on a fair scheduling base. However, the question arises if a further improvement is possible. As seen in Section 5.3 is the mean throughput result per wireless terminal the mean of the throughput results for the single priority classes. In the last Section, 5.4, we can see for the simple RSSA, that the mean throughput values for the upper two priority classes are quite high whereas the mean throughput for the last priority class is equivalent to the mean throughput of the SSA and therefore low compared to the other priority classes. For the overall mean throughput this means as consequence that improvement of the throughput can only be achieved by increasing the throughput of the last priority class. In order to discuss this matter, let us first consider the distribution of the the sub-carriers as assigned by the access point to the different priority classes. In particular we are interested in the weight distribution of these assigned sub-carriers. Therefore the question is for the specific setting assumed in Section 5.4, how many sub-carriers of weight one were assigned to priority class one, how many of weight two and how many of weight three. Figure 5.4 shows the sub-carrier weight distribution as obtained by the value of  $p_g = 0.9$ , therefore this Figure is independent of the scheduling policy just gives us the distribution resulting from the statistical model assumed.

If we compare now how the single priority classes are assigned sub-carriers with certain weights, we observe the following. Since the first priority has no restrictions at all, it is assigned its sub-carriers quite similar to the shown distribution of Figure 5.4, with the difference that it never is assigned a sub-carrier with weight zero. As stated, the first priority class will be assigned the first  $N$  good sub-carriers out of  $S$  sub-carriers total (Figure 5.5). Therefore the ratio between the weights this priority class is assigned will remain the same to the ratio resulting from the statistical model, except for the weight zero sub-carriers, which are not assigned at all. Instead, more sub-carriers with weights bigger than zero will be

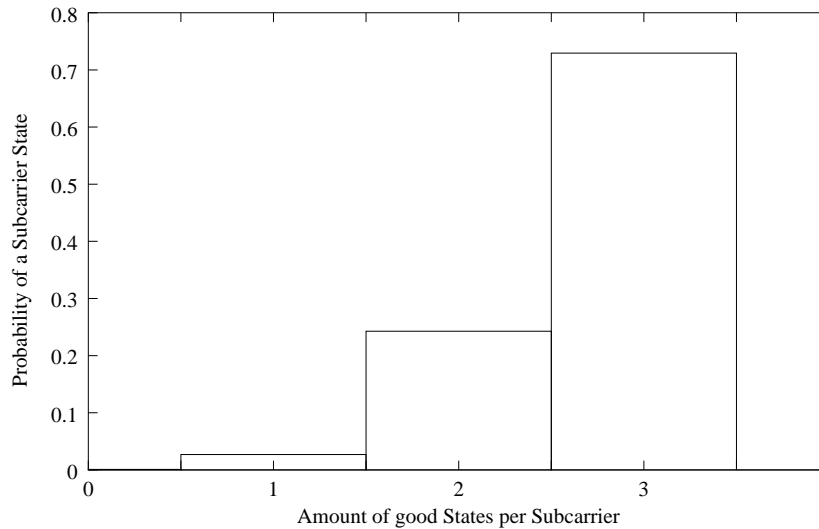


Figure 5.4: Distribution of Sub-Carrier Weights for  $p_g = 0.9$

assigned. The same is true for the second priority class (Figure 5.5). It also is assigned a lot weight three sub-carriers in order to get almost no weight zero sub-carriers. Due to the lower probability of obtaining  $N$  good sub-carriers, this priority class will receive some weight zero sub-carriers. But for the third priority class this means, that a lot of weight three and weight two sub-carriers are already assigned, leaving it up with more weight one sub-carriers (Figure 5.5). This leads to the situation, that these sub-carriers with weight one could be used by other priority classes, but often not by the last priority class. It reduces the throughput of the last priority class. Since this class is the only one that can be utilized for an even better throughput, one approach is to avoid a situation where the access point encounters a lot of weight one sub-carriers to assign to the last priority class. The distribution of the sub-carrier weights chosen by the single classes are shown in Figure 5.5.

The simplest way to influence the sub-carrier weight distribution the priority classes get assigned is to force the access point always first to assign the lowest sub-carrier weights available and being in state  $G$  towards the actual priority class. Therefore the highest priority would first get assigned all sub-carriers that are in a good state towards him and furthermore have the weight one. Next, if he does not have already  $N$  sub-carriers assigned, the access point will choose out of the sub-carriers with the weight two. At last, the access point will assign sub-carriers with the weight three to the actual priority class. This also applies to priority class two. After simulating such a policy the result is indeed, that priority class one obtains almost twice as much sub-carriers with weight one, even more than twice as much sub-carriers with weight two and roughly only half the amount of sub-carriers with weight three (Figure 5.6). For class two, things almost stay the same as in the original simple RSSA, beside the fact that weight one sub-carriers are assigned twice as much (Figure 5.6). Class three instead is assigned now a lot more weight three sub-carriers, which improves the overall throughput. We will refer to this scheduling policy as the advanced RSSA. The new weight distribution plots are shown in Figure 5.6.

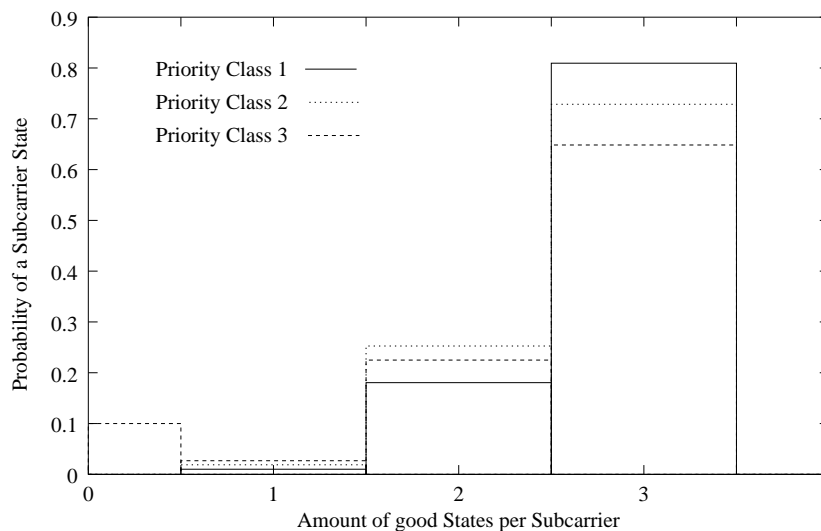


Figure 5.5: Distributions of chosen Sub-Carrier Weights by Priority Classes for the simple RSSA with  $S = 12$ ,  $J = 3$ , and  $p_g = 0.9$

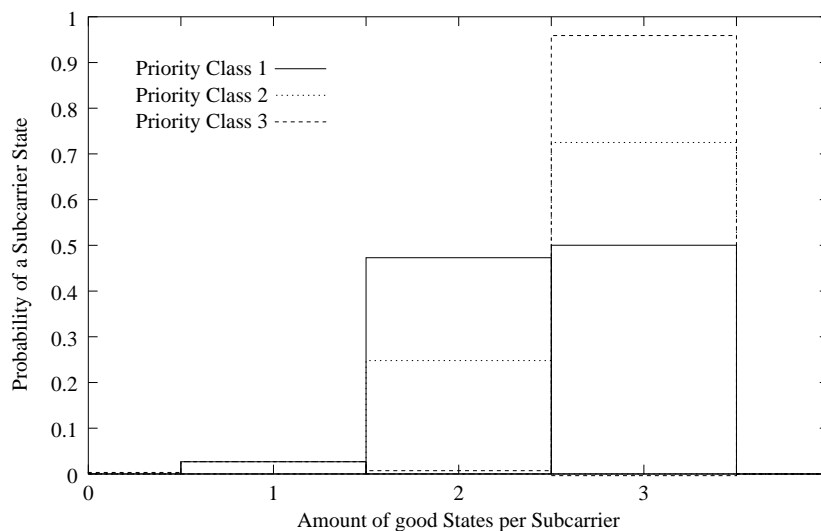


Figure 5.6: Distributions of chosen sub-carriers by Priority Classes for the advanced RSSA with  $S = 12$ ,  $J = 3$ , and  $p_g = 0.9$



Evaluating the throughput of this scheduling policy, we encounter some difficulties. The choosing function at the access point depending on the weights of the sub-carriers makes it very difficult to derive theoretically the overall throughput for the algorithm. Therefore, we have to consider throughput results obtained by simulating the algorithm as done in Section 5.4. By doing this with the known parameters from Section 5.4, we obtain the following results:

$$D_{first} = 4 \cdot \frac{b}{T_s} \quad D_{second} = 4 \cdot \frac{b}{T_s} \quad D_{third} = 3.98 \cdot \frac{b}{T_s}$$
$$\Rightarrow D_{advanced\ RSSA} = 3.9933 \cdot \frac{b}{T_s} = \frac{31.946}{T_s} \text{bits}$$

This corresponds to a ratio of 0.9983, comparing the actual throughput result with the throughput of the wireless terminal in the case of the perfect sub-carriers. In other words, although our sub-carrier state model includes that sub-carrier sometimes can not convey a symbol for the period of a symbol time, the advanced RSSA achieves a throughput that is almost similar to a FDMA setting where sub-carrier never fail in conveying a symbol.

The same arguments are true for the advanced RSSA regarding the number of  $J$  and the number of  $S$ . An increase of the wireless terminals  $J$  while still assuming a constant  $N$  will increase the throughput results per wireless terminal, while the same is true for an increasing number of  $N$  while  $J$  remains constant.

## 5.6 Upper Throughput Limits of Scheduling Policies for the Binary Sub-Carrier State Model

So far our performance measure for the scheduling policies was always the ratio between the throughput actually obtained and the throughput for a static sub-carrier assignment where sub-carriers are always in state  $G$  (perfect sub-carrier throughput, see Section 5.4). Although the concept of perfect sub-carriers represents the absolute upper throughput limit, it is obvious that it does not represent the upper throughput limit for the binary sub-carrier state model assumed in this section. Clearly, the upper throughput limit for the binary sub-carrier state model will be lower than the throughput in the perfect sub-carrier case.

A more precise throughput limit is desirable, since it can judge better if the developed scheduling policies work efficient. Of course the performance measure used so far is sufficient in ranking different scheduling policies among each other. But it does not tell us, if there is still a potential improvement, i.e. if there is still a better scheduling policy achievable. Therefore an upper throughput limit derived for this sub-carrier model yields us more information about the efficiency of our scheduling policies developed so far.

In the case of the binary sub-carrier state model the upper throughput limit is quite easy to obtain. An optimal policy would use all sub-carriers per transmission time that do not have the weight zero. It is important to note here the trade-off between optimal throughput, fairness and complexity. A scheduling policy assigning always all sub-carriers towards wireless terminals that have at least the weight one might not be fair. If it is fair, it might be very complex. Nevertheless its throughput will be optimal. Let us assume her for the optimal policy, that it provides fairness.

Algorithm	Norm. Throughput
SSA	0.901
PSA	0.906
Simple RSSA	0.967
Advanced RSSA	0.9993

Table 5.2: Normalized Throughput Results for all Scheduling Policies of the binary Sub-Carrier State Model

Now let us obtain the mean throughput for such a scheduling policy. According to the assumptions of the binary sub-carrier state model we obtain for the probability that a sub-carrier has a weight of zero :

$$P(\text{A sub-carrier is in state } B \text{ towards all three wireless terminals}) = (1 - p_g)^J$$

Therefore the probability that a sub-carrier has at least a weight of one will be:

$$P(\text{A sub-carrier has at least the weight one}) = 1 - (1 - p_g)^J \quad (5.8)$$

For the example setting that was investigated in 5.4, we obtain for the probability that a sub-carrier might be assigned, since he has at least the weight one:

$$P(\text{A Sub-Carrier has at least the weight one}) = 0.999$$

The value resulting from Equation 5.8 represents the probability that a sub-carrier is usable towards a certain wireless terminal. Multiplied with the amount of sub-carriers being assigned towards each wireless terminal per transmission symbol time and being multiplied by the symbol length, this will result in an upper throughput limit for the binary sub-carrier state model (Equation 5.9):

$$\begin{aligned} D_{\text{optimal Throughput}} &= P(\text{A Sub-Carrier has at least the weight 1}) \cdot N \cdot b \\ &= (1 - (1 - p_g)^J) \cdot N \cdot \frac{b}{T_s} \quad (5.9) \end{aligned}$$

If we now consider the ratio between the actual scheduler throughputs resulting from the policies discussed so far and the optimal throughput achievable, we will obtain a ratio that was already formally introduced as normalized throughput measure in Section 4. Table 5.2 holds the normalized throughput values for all scheduling policies mentioned so far.

Although the efficiency of the advanced RSSA was quite obvious from the absolute throughput values compared to the throughput values obtained in the case of perfect sub-carriers, we can deduce from the normalized throughput value for the advanced RSSA, that it almost always uses all sub-carriers that potentially can convey a symbol. By doing so, it still guarantees fairness and has an acceptable complexity.

## 5.7 Comparison of the Policies for a variable Parameter $p_g$

So far we have discussed the mean throughput results for a varying  $N$  and a varying  $J$  for each scheduling policy. However, the question arises how the policies behave if the parameter  $p_g$  is changed. We will investigate the behavior of all dynamic scheduling policies presented so far and compare it to the behavior of the SSA for this case. The basic values assumed here are  $J = 3$ ,  $N = 4$ , and  $b = 8$  bits. For the results of the advanced RSSA we simulated the throughput values. The other policies throughput results were obtained by the analytical formulas. The value of  $p_g$  was varied between zero and one with a granularity of  $\frac{1}{100}$ , i.e. starting at a probability of  $p_g = 0.01$  and ending at a probability of  $p_g = 0.99$ . Figure 5.7 shows the absolute throughput value curves for each scheduling policy and also includes the theoretical upper limit as discussed in 5.6. Figure 5.8 shows the normalized throughput results, also according to Section 5.6.

From Figure 5.7 several results can be drawn. As the throughput of the SSA increases linearly, the throughput of all other policies does not. Instead, within certain ranges of the probability  $p_g$  the throughput of some scheduling policies does increase stronger while within other ranges it does not.

For the advanced RSSA it is interesting to observe, that the throughput results increase almost identical to the behavior of the upper limit. This is also indicated by the advanced RSSA curve of Figure 5.8. The normalized throughput is almost always very close to one. However, for values around  $p_g = 0.4$  the normalized curve has a minimum. Therefore, in terms of efficiency, the advanced RSSA works here worst. The reason for this minimum is not quite clear. The authors have the suspicion, that for the area around the minimum normalized throughput of the advanced RSSA probability ratios of the weight of sub-carriers could be the reason for the local inefficiency. If too many weight one sub-carriers are available compared to the weight two and three amounts, the advanced RSSA can maybe not find an optimal assignment which is also fair. However, it was not possible to find for the ratios between the sub-carrier weight probabilities any minimum corresponding to the minimum of the normalized throughput of the advanced RSSA. Compared to all other dynamic policies the advanced RSSA outperforms them at all values of  $p_g$ .

For the simple RSSA it is important to note that in both Figures the curves were obtained by simulating the algorithm. As expected does the simple RSSA not provide such a good performance as the advanced RSSA does, but it still outperforms the PSA and the SSA for all probability values. Also in terms of efficiency does the simple RSSA behave well compared to the PSA and the SSA. Interestingly, the difference in terms of throughput to the advanced RSSA becomes significant the higher the probability of  $p_g$  is. This is due to the fact, that the higher  $p_g$  is, the bigger is the probability, that a sub-carrier might have a weight of three. Therefore the strategy of the advanced RSSA becomes more and more effective.

In order to judge the quality of the derived lower limit for the simple RSSA, Figure 5.10 holds the absolute throughput values of the simulated and analytical derived values for the varying parameter  $p_g$ . As it is shown, the simulated and derived values correspond quite good for high values of  $p_g$ . However, for low values of  $p_g$  the lower limit does not match with the results of the simulation any more. The achieved throughput is much smaller. This is clearly due to the fact, that in the case of the analytical result for higher priority classes only the case of obtaining  $N$  good sub-carriers is included (refer also to Section 5.3). For a low

$p_g$  the probability for a higher priority class to obtain such a set of sub-carriers is decreased obviously. Therefore the analytical result differs more and more from the simulated result the lower the probability of  $p_g$  gets. Figure 5.11 shows this behavior also in terms of the normalized throughput.

For the PSA we draw the following results. For a relative low probability  $p_g$  the PSA behaves quite good in terms of absolute and relative throughput. However, as the probability that a sub-carriers has the weight one reaches a maximum and furthermore decreases constantly (Figure 5.9), the normalized throughput decreases. The minimum value for the PSA normalized throughput is at  $p_g = 0.55$ . For higher values of  $p_g$  the absolute throughput (and the normalized too) of the PSA becomes more and more equivalent to the absolute throughput of the SSA. This is due to the fact, that for a relative high probability  $p_g$  the defined switching rule of the PSA only rarely switches sub-carriers. Instead, the throughput is more and more dominated by the throughput of the initially assigned fixed sub-carrier set and therefore corresponds more and more to the SSA results.

The behavior of the SSA was already discussed in Section 5.1. As we can see also from the Figure does the absolute throughput of the SSA depend linearly on the probability  $p_g$ .

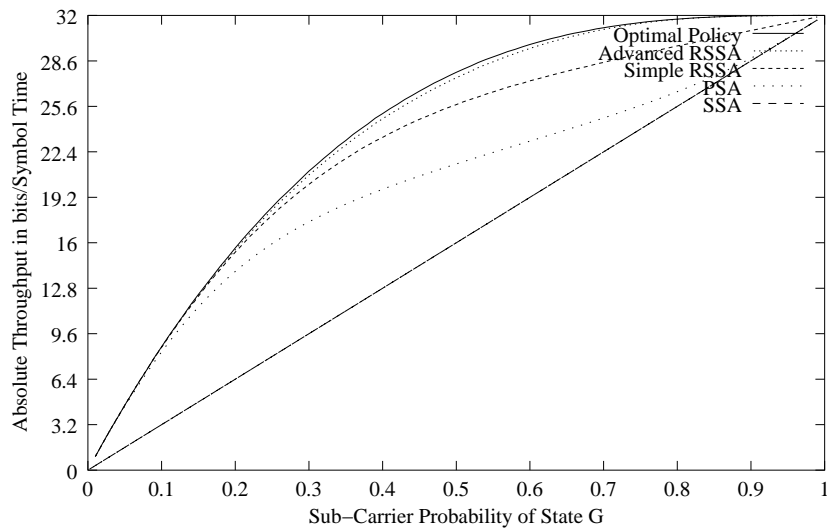


Figure 5.7: Absolute Throughput Values for all presented Scheduling Policies for a varying Sub-Carrier State Probability  $p_g$  with  $S = 12$ ,  $J = 3$  and  $N = 4$

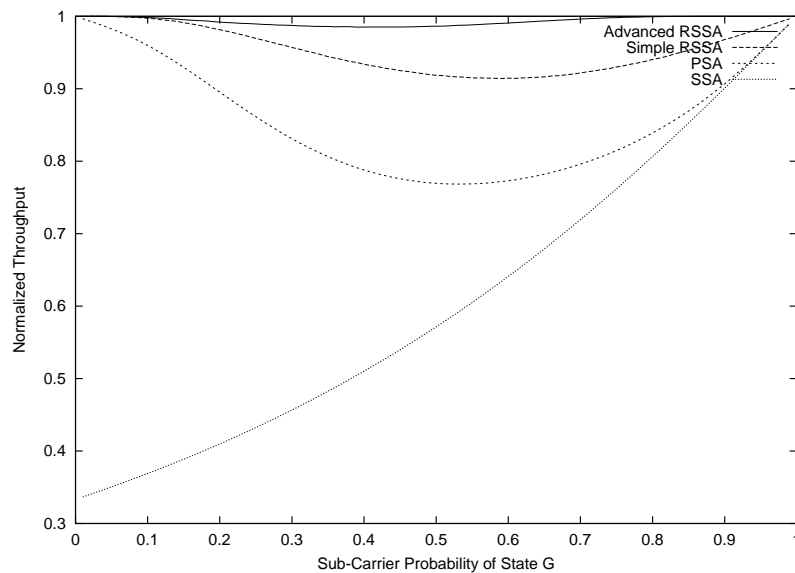


Figure 5.8: Normalized Throughput Values for all presented Scheduling Policies for a varying Sub-Carrier State Probability  $p_g$  with  $S = 12$ ,  $J = 3$  and  $N = 4$

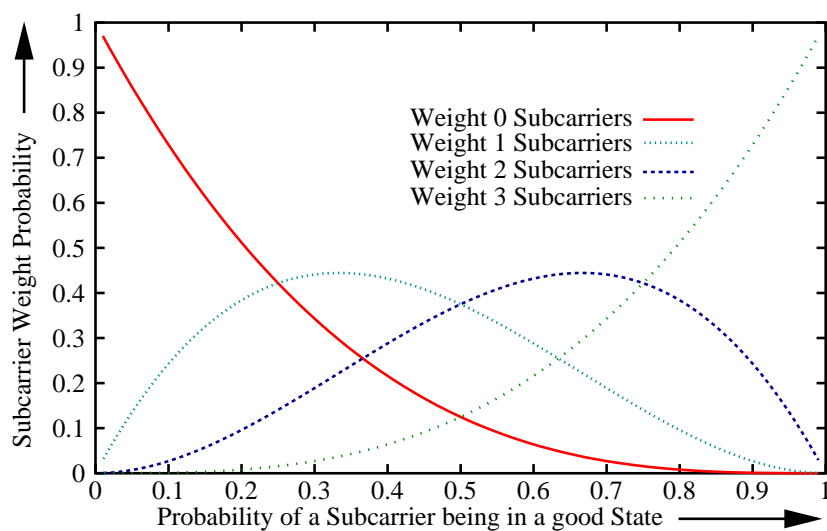


Figure 5.9: Sub-Carrier Weight Probabilities for a varying Sub-Carrier State Probability  $p_g$  with  $J = 3$

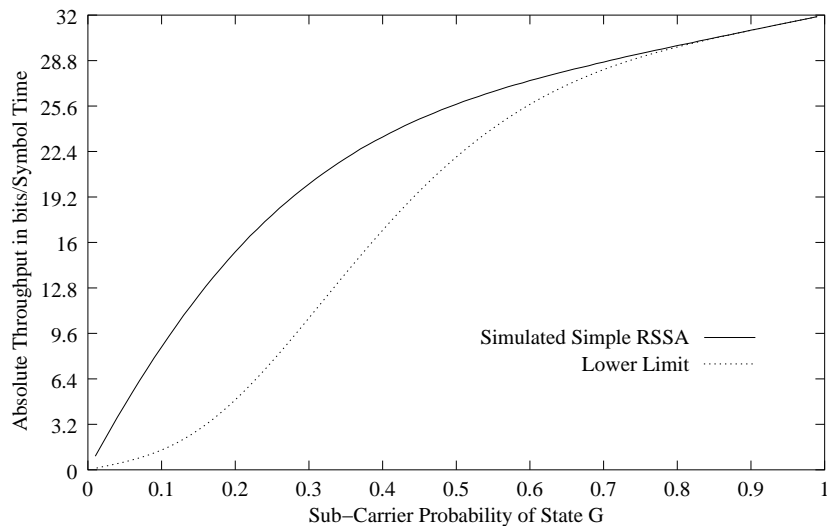


Figure 5.10: Absolute Throughput Comparison between the simulated simple RSSA and the analytical Lower Limit for a varying Sub-Carrier State Probability  $p_g$  with  $S = 12$ ,  $J = 3$  and  $N = 4$

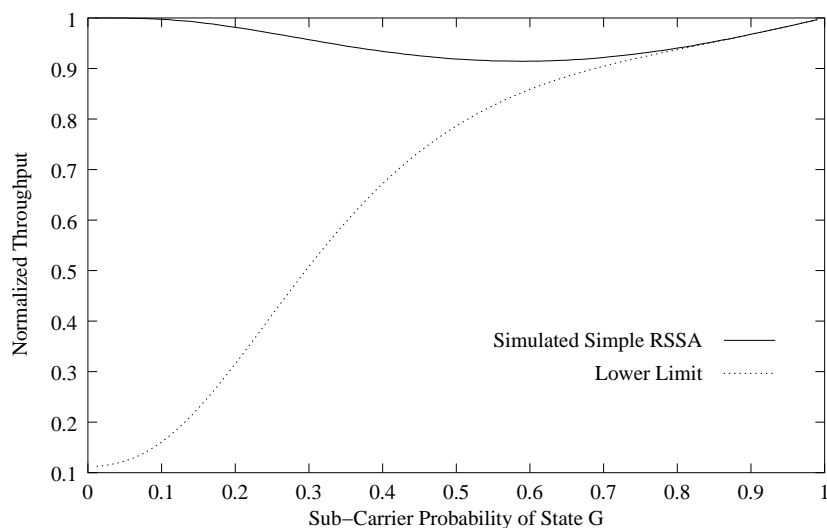


Figure 5.11: Normalized Throughput Comparison between the simulated simple RSSA and the analytical Lower Limit for a varying Sub-Carrier State Probability  $p_g$  with  $S = 12$ ,  $J = 3$  and  $N = 4$

## 5.8 Results for the Binary Sub-Carrier State Model with non-perfect Estimates

Up until now we always assumed that the access point has a perfect knowledge of the sub-carrier states within the next symbol time interval. Of course in reality we almost never encounter such a situation. Instead sub-carrier states will be different in a certain amount of cases than they were predicted. In this Section we desire to know, which impact this will have on our throughput results. Therefore we first have to add some assumptions regarding the failing of estimates to our model.

Let us assume the following case. For each estimate of a sub-carrier state towards some wireless terminal for the next symbol time, there exists a probability that this prediction will in fact become true. We can think of this probability as of the reliability of each estimate. Let us further assume that this probability is equal for all estimates. Let us denote this probability by  $p_r$ . Therefore out of an arbitrary amount of estimates  $x$ ,  $(1 - p_r) \cdot x$  sub-carrier states will not correspond to their predictions.

Since all scheduling policies try to convey a symbol through the assigned sub-carriers to the wireless terminals only if the predicted state is  $G$ , the throughput values will be reduced by the probability that the sub-carrier estimates are reliable. Therefore the resulting mean throughput values will be for all policies:

$$D_{\mathbf{x}, \text{ non-perfect Estimates}} = p_r \cdot D_{\mathbf{x}, \text{ perfect Estimates}} \quad (5.10)$$

Clearly, the above mentioned rule that symbols are only conveyed on sub-carriers which were estimated good, decreases the possible throughput in the case of non-perfect estimates. If the access point would convey symbols even in the case that a sub-carrier was estimated to be in a bad state, the above shown throughput would be higher. The reason for this is that in the case of unreliable estimates of course also bad estimated sub-carriers could turn out to be good. This would be the case with a probability of  $1 - p_r$ . Therefore, if the access point would also try to convey symbols on sub-carriers being estimated bad, the throughput would be increased. The exact throughput is given by:

$$D_{\mathbf{x}, \text{ non-perfect Estimates}} = p_r \cdot D_{\mathbf{x}, \text{ perfect Estimates}} + (1 - p_r) \cdot \left( \frac{N \cdot b}{T_s} - D_{\mathbf{x}, \text{ perfect Estimates}} \right) \quad (5.11)$$

The additional term in Equation 5.11 results from the extension to the access point scheduling possibilities. Out of all bad sub-carriers (which contribute to the throughput of  $\frac{N \cdot b}{T_s} - D_{\mathbf{x}, \text{ perfect Estimates}}$ ) each sub-carrier will be good with the probability of  $1 - p_r$  and therefore surprisingly convey a symbol.



## Chapter 6

# Conclusions

Several results can be drawn from the project outcomes so far.

- For the rather simple statistical sub-carrier model assumed the RSSA scheduling policy increases the throughput compared to the throughput of the static assignment scheme. The throughput of the simple RSSA assignment scheme can be even further increased if we introduce a choosing function which assigns sub-carriers with respect to sub-carrier states towards other priority class wireless terminals. In the case of the advanced RSSA it is possible to completely hide the effect of noise encountered for each sub-carrier from the wireless terminals. By this, throughput values become equal with the ones of a noiseless sub-carrier case.
- The normalized throughput values of all dynamic scheduling policies increases for cases with more wireless terminals per cell or more sub-carriers being available. The reason for this was discussed at several points within this report. The static scheduling policy shows an increase of the absolute throughput but not of the normalized throughput. This may be interpreted as benefit due to using the sub-carrier state knowledge provided by the physical layer. The more information may be used, the bigger the benefit is.
- However, the downside of dynamic scheduling policies is the signaling overhead required to indicate the new set of sub-carriers assigned towards all wireless terminals. As the amount of sub-carriers increases as well as the amount of wireless terminals increases, signaling increases too. This is of course not true for the static scheduling policy.
- All results here assume scheduling on a per symbol base. If one considers a scenario where sub-carrier states are assumed to be stable for a longer time period, say the transmission time of a packet, these results can also be interpreted as scheduling throughput results on a per packet base. Therefore if scheduling per symbol might get to complex due to signaling aspects, these results also apply to scheduling on a per packet base.
- Here we do not assume any correlation behavior of the sub-carrier states between each other. While this might be realistic for states of one sub-carrier towards different wireless terminals, this is certainly not true for sub-carrier states of close sub-carrier bands towards the same wireless terminal. Such a state correlation does not necessarily decrease the mean throughput, but could call for modifications of the advanced RSSA

version. In fact, it could lead to a complexity decrease, since a sub-carrier state could represent whole sub-carrier blocks rather than a single sub-carrier.

- In this report we considered a statistical model which included two states possible for each sub-carrier towards each wireless terminal. However, in more realistic models one should consider more states being possible per sub-carrier towards each wireless terminal. The investigation of such models has already been performed and is documented in [6].

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