Tutorial “Performance Evaluation Techniques” First Problem Sheet

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Problem 1:
Suppose there are $N = 10$ urns behind a curtain, such that you cannot see them. The urns are numbered from $i = 1, \ldots, 10$. Urn i contains ten balls: i white balls and $10-i$ red balls. A person behind the curtain picks one urn at random (all urns are equiprobable), picks ten balls with replacement from this urn and notes the result. Afterwards, the person tells you that $N_W = 3$ white balls were drawn (and accordingly $10 - 3 = 7$ red balls). What is the probability that the person has chosen urn $i \in \{1, 2, \ldots, 10\}$? Please give a derivation and numbers.
(Hint: Bayes theorem, law of total probability, binomial distribution)

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Problem 2:
Be $X \sim \mathcal{N}(\mu, \sigma^2)$ a random variable having normal distribution with mean value $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$. The density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad \text{and we have} \qquad \int_{-\infty}^{\infty} f(x)\, dx = 1$$

for $x \in \mathbb{R}$. Show without integration that the moment generating function $\mathcal{M}_X(\theta)$ is given by:

$$\mathcal{M}_X(\theta) = E\left[e^{\theta X}\right] = e^{\theta\mu + \frac{\theta^2\sigma^2}{2}} = \exp\left(\theta\mu + \frac{\theta^2\sigma^2}{2}\right)$$

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Problem 3:
Be X a random variable with gamma distribution. The gamma distribution has the density function:

$$f(x) = \frac{\lambda(\lambda x)^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$$

for $x \geq 0$ with $\lambda > 0$ and $\alpha > 0$ being parameters, and $\Gamma(x)$ being the well-known *Gamma function*:

$$\Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx$$

(which is like the factorial function for real numbers: for $n \in \mathbb{N}$ we have $n! = \Gamma(n+1)$).

1. Compute the moment generating function $\mathcal{M}_X(\theta)$. (Hint: use clever substitution and the definition of the Gamma function).

2. For $\alpha = 1$ the Gamma distribution degenerates into an exponential distribution. For $\alpha > 1$ being an integer, compare the resulting $\mathcal{M}_X(\theta)$ with that of the exponential distribution ($\alpha = 1$) and give an interpretation.

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Problem 4:
Your company runs a large website. Technically, the website is driven by a number of servers and a load-balancer "in front" of the system. The load balancer accepts a request from outside (all requests arrive to the same network interface of the load-balancer), chooses a server randomly and forwards the request to this server. Customers call your boss and complain about slow response times. Your boss asks you how to improve the system. It is your task to develop a recommendation based on a performance evaluation.

This is an "open" problem, there is no "right" or "wrong" solution; use your imagination. You should at least:

- clearly define the system under study, identify important system components / subsystems and their most important attributes (example: the load balancer has a queue for yet unprocessed requests, and this queue is finite)
- define performance measures
- identify important input variables (e.g. queue length of the load balancer, types of web pages [static vs. dynamic], bandwidth of load balancers incoming interface etc.)
- decide on the factors and their levels
- Propose measurement or simulation setups studying these measures
- decide on an experimental design: which factor / level combinations should be investigated?

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