This document, for information, presents a new method for deriving the transmission rating factor $R$ from MOS-values in closed form using a slightly modified version of the transformation formula to be found in ITU-T Rec. G.107 (2000). The formula was found and brought to our attention by Mr. Christian Hoene from the Telecommunication Networks Group at the Technical University Berlin, Germany. It is addressed to Question 8 in order to decide whether it will be useful to include such a formula in future versions of G.107, e.g. as an Appendix which might be useful for applying Rec. P.833 and draft new Rec. P.DIEM.

Introduction

The E-Model (ITU-T Rec. G.107) is the recommended computational model to be used for transmission-planning. The E-Model predicts the impact of typical impairments on speech quality. The primary output is the “Rating Factor” $R$. It ranges from 0 (worst) to 100 (best). The Mean Option Score (MOS) can be obtained from the $R$ Factor with a converting formula. MOS has a scale from one (bad) to five (excellent) and is used for subjective and objective evaluations of perceived speech quality.

In this report we describe an unexpected behavior of the converting formula given in G.107 for low $R$ values. As a consequence, the formula cannot be inverted to calculate the $R$ factor from a given MOS value. Thus, we redefine the converting formulae and present its inverted form, which allows $R$ factors to be calculated from MOS values.
New Conversion Formula

The Mean Option Score (MOS) can be obtained from the R-Factor as described in ITU-T Rec. G.107 (2000) (Equation B-4).

\[
MOS = \begin{cases} 
R \leq 0 : & 0 \leq R \leq 100 : \left(1 - \frac{7}{1000} R + \frac{3}{6250} R^2 - \frac{7}{1000000} R^3\right) \\
R \leq 100 : & 0 \leq R \leq 100 : \frac{1}{4.5} 
\end{cases}
\]  

This formula calculates for each \( R \)-value between 0 and \( 80 - 30\sqrt{6} \approx 6.5 \) an MOS value below 1 (see Figure 1). Thus, the formulae cannot be inverted (as stated also in ITU-T Rec. P.833, 2001) to calculate - even within the given range of R and MOS - one R factor from a given MOS value.

Therefore, we redefine the converting formula to:

\[
MOS = \begin{cases} 
R \leq 6.5 : & 0 \leq R \leq 100 : \left(1 - \frac{7}{1000} R + \frac{3}{6250} R^2 - \frac{7}{1000000} R^3\right) \\
6.5 \leq R \leq 100 : & \frac{1}{4.5} 
\end{cases}
\]  

This equation can be inverted with the Candono’s Formula:

\[
R = \frac{160}{3} \cdot \frac{5}{21} \left(\sqrt[3]{p + iq} + \sqrt[3]{p - iq}\right) 
\]

with the MOS range between 1 and 4.5 and

\[
q = 1470 \sqrt{-903522 + 1113960 MOS - 202500 MOS^2} \\
p = 1819468 - 661500 MOS \\
p, q > 0
\]

The equation requires computations with complex numbers. However, the result \( R \) is a real value in any case. Thus, we simplify the term to:
\[ R = \frac{20}{3} \left( 8 - \sqrt{226 \cos \left( h + \frac{\pi}{3} \right)} \right) \]  

(5)

with

\[ h = \frac{3}{4} \arctan \left( 18566 - 6750MOS, 15\sqrt{-903522 + 1113960MOS - 202500MOS^2} \right) \]  

(6)

Using these equations, MOS values can be converted to \( R \) factors (see Figure 2).

**Summary**

The Mean Opinion Score MOS can be calculated from the transmission rating factor \( R \) via:

\[
MOS = \begin{cases} 
R \leq 6.5 : & 1 \\
6.5 \leq R \leq 100 : & 1 - \frac{7}{1000} R + \frac{7}{6250} R^2 - \frac{7}{1000000} R^3 \\
R \leq 100 : & \frac{1}{4.5} 
\end{cases}
\]  

(2)

\( R \) can be calculated from MOS using the formula

\[ R = \frac{20}{3} \left( 8 - \sqrt{226 \cos \left( h + \frac{\pi}{3} \right)} \right) \]  

(5)

with

\[ h = \frac{3}{4} \arctan \left( 18566 - 6750MOS, 15\sqrt{-903522 + 1113960MOS - 202500MOS^2} \right) \]  

(6)