

## On the Signaling Overhead in Dynamic OFDMA Wireless Systems\*

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**Abstract:** It is well known that dynamic OFDMA systems potentially increase the spectral efficiency of wireless systems. They exploit diversity effects in time, space, and frequency by assigning system resources periodically to terminals. Informing the terminals about new assignments creates a signaling overhead. So far, simple signaling schemes have been presented in order to quantify the cost of signaling for such systems. However, by exploiting the correlation of sub-carrier attenuations in time, it is possible to decrease the cost of signaling significantly. In this paper we investigate such a scheme based on including the signaling cost in the optimization problem of sub-carrier assignments.

### 1 Introduction

*Orthogonal Frequency Division Multiplexing* (OFDM) appears to be quite an attractive transmission scheme in order to overcome the impairments of wireless broadband channels. In OFDM systems, the total bandwidth is split into many *sub-carriers* [vNP00]. Various standards for wireless systems are already based on the OFDM transmission scheme; it is also a strong candidate as transmission scheme for future cellular wideband communications systems (e.g. 4G systems).

The attenuations of sub-carriers to the different terminals exhibit a strong diversity in frequency, time, and space. A *dynamic OFDMA* [WCLM99] link layer concept exploits this diversity to achieve a higher spectrum utilization. The idea is to assign those sub-carriers to the respective terminals for which the current channel conditions are best. The sub-carriers are re-allocated periodically in order to adapt to the time-varying channel attenuations. This implies solving an optimization problem to find the currently best assignments. It is well known that a dynamic OFDMA approach can significantly improve system metrics such as throughput or transmit power.

There is obviously a *signaling overhead* in dynamic OFDMA systems. The receiving terminals need to know which sub-carriers are assigned to them. This affects the system's performance. The impact of the signaling overhead on system performance assuming a signaling phase of *fixed length* has been studied in in [GPKW04]. Sub-carrier attenuations are, however, correlated over time, so few assignments may change from one down-link phase to the next. To exploit this correlation, we study a more flexible signaling model in which the signaling phase has variable length and only assignment changes are transmitted. In this model, we then assign sub-carriers such as to optimize *net* throughput. This leads to a significantly more complex optimization problem, for which we develop a solution algorithm. We show that in case of a high correlation in time, this scheme pays off in terms of the resulting throughput.

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## 2 System Model and Gross Throughput

Consider the down-link of a single cell of a wireless system. The access point serves  $J$  terminals by sending a continuous stream of data to each of them. We consider a slotted system in which time is divided into units (frames) of duration  $T_f$ . A total bandwidth of  $B$  [Hz] at the center frequency  $f_c$  is available for data transmission. For this band, a maximum total transmit power of  $P_{\max}$  may be emitted. The given bandwidth is split into  $N$  sub-carriers. Although the symbol duration  $\frac{N}{B} = T_s$  is equal for each sub-carrier, a different amount of bits might be represented per symbol for each sub-carrier. A total of  $M$  modulation types are available.

The perceived signal quality per sub-carrier, i.e. the SNR, varies from frame to frame. The instant SNR of sub-carrier  $n$  for terminal  $j$  at time  $t$  is given by  $v_{j,n}^{(t)} = p_n^{(t)} \cdot (h_{j,n}^{(t)})^2 / \sigma^2$ . Here  $p_n^{(t)}$  denotes the transmission power,  $h_{j,n}^{(t)}$  denotes the attenuation of sub-carrier  $n$ , and  $\sigma^2$  is the noise power. The attenuation varies due to path loss, shadowing and fading. It is assumed to be constant per frame.

Each frame is split into a down-link and an up-link phase. OFDMA is applied during the down-link phase, which has a duration of  $T_d$ . Prior to each down-link phase, the access point generates new assignments of sub-carriers to terminals based on the knowledge of the sub-carrier states. We assume perfect channel knowledge at the access point. Binary variables represent the assignment decisions, i.e.  $x_{j,n}^{(t)} = 1$  if terminal  $j$  is assigned sub-carrier  $n$  for down-link phase  $t$ ;  $= 0$  otherwise. Each sub-carrier can be assigned to at most one terminal at a time. This yields a linear constraint:

$$\sum_j x_{j,n}^{(t)} \leq 1 \quad \forall n . \quad (1)$$

We assume transmit power to be evenly spread on the sub-carriers. The SNR for different sub-carriers thus varies according to the attenuations. These variations are exploited using adaptive modulation. Given the current sub-carrier attenuations, a potential throughput in bit per sub-carrier/terminal pair can be calculated for the following down-link phase. The attenuation matrix with values  $h_{j,n}^{(t)}$  can thus be converted into a *bit matrix* with values  $b_{j,n}^{(t)}$ . Given a certain assignment realization, the total number of transmitted bits  $e$  in the next down-link phase  $t$  is given by

$$S \sum_{j,n} b_{j,n}^{(t)} \cdot x_{j,n}^{(t)} . \quad (2)$$

This overall data amount per down-link phase serves as our metric, which we want to maximize. However, this is a *gross* value as it does not contain the performance loss due to signaling.

For avoiding starvation of terminals far away from the access point, we fix a maximum number  $l_j$  of sub-carriers that each terminal  $j$  can obtain. This can be formulated as a linear constraint:

$$\sum_n x_{j,n}^{(t)} \leq l_j \quad \forall j . \quad (3)$$

Based on the notation introduced, the maximization of *gross* throughput in the dynamic OFDMA system can be formulated as a linear integer programming optimization problem, referred to as (PLAIN).

$$\max \quad S \cdot \sum_{j,n} b_{j,n}^{(t)} \cdot x_{j,n}^{(t)} \quad \text{s.t.} \quad (1), (3) . \quad (\text{PLAIN})$$

In graph theory, (PLAIN) is called the *bipartite weighted b-matching problem*. In the present case, it can be shown that it has a complexity of at most  $O(N^3)$  using regular matching algorithms [KV00]. In practice, the optimal solutions to this problem can be computed within milliseconds for the problem sizes considered here using linear programming algorithms.

### 3 Maximizing Net Throughput for the Variable Signaling Model

We consider an *inband signaling scheme*. In addition, we assume the signaling information to be *broadcasted* during the signaling phase using a predefined, fixed modulation type with bit rate  $b_{\text{sig}}$ , the information is broadcasted using all  $N$  sub-carriers. Broadcasting the signaling information adds to the system's flexibility; terminals can easily receive arbitrary numbers of sub-carriers without losing synchronization to the dynamic assignments.

The triple of information (sub-carrier, terminal, modulation) has to be transmitted for each assignment during the signaling phase. In [GPKW04] a simple signaling scheme was introduced. In each down-link phase, all assignments of sub-carriers are signaled in a predefined order. Therefore, a fixed number of  $N \cdot (\lceil \log_2(J) \rceil + \lceil \log_2(M) \rceil) / N \cdot b_{\text{sig}}$  signaling bits is required per down-link phase leading to  $\lceil (\lceil \log_2(J) \rceil + \lceil \log_2(M) \rceil) / b_{\text{sig}} \rceil$  OFDM symbols of signaling overhead. This scheme is referred to as fixed-size signaling field model.

In order to save overhead, it is possible to transmit only the assignments of sub-carriers that have changed with respect to the last down-link phase. Signaling an assignment change from one down-link phase to the next one consumes  $C_{\text{sig}} = \lceil \log_2(N) \rceil + \lceil \log_2(J) \rceil + \lceil \log_2(M) \rceil$  bits. If only a few assignments change, the overall cost for signaling can be reduced significantly. There is a clear trade-off between exploiting diversity by dynamically reassigning sub-carriers and avoiding reassignments in order to reduce signaling overhead. We thus propose a state-aware scheme for the access point to decide which dynamic reassignments pay out in terms of *net throughput*. Knowing the assignments for down-link phase  $t - 1$ , the cost for assigning sub-carrier  $n$  to terminal  $j$  in phase  $t$  is  $c_{j,n}^{(t)} = C_{\text{sig}}$  if  $x_{j,n}^{(t-1)} = 0$  and  $c_{j,n}^{(t)} = 0$  otherwise. During the signaling phase, a fixed modulation type with bit rate  $b_{\text{sig}}$  is used. The number of OFDM symbols used for signaling is thus  $\varsigma = \lceil \sum_{j,n} c_{j,n}^{(t)} \cdot x_{j,n}^{(t)} / N \cdot b_{\text{sig}} \rceil$ . Maximizing *net throughput* thus corresponds to:

$$\begin{aligned}
 \max \quad & (S - \varsigma) \cdot \sum_{j,n} (b_{j,n}^{(t)} \cdot x_{j,n}^{(t)}) \\
 \text{s.t.} \quad & (1), (3) \\
 \varsigma \geq & \frac{\sum_{j,n} c_{j,n}^{(t)} \cdot x_{j,n}^{(t)}}{N \cdot b_{\text{sig}}} \\
 & \varsigma \text{ integral.}
 \end{aligned} \tag{VAR OPT}$$

This problem is non-linear as the objective function includes the product of variables. In our context there are very few possible values that  $\varsigma$  can actually take. Their enumeration is therefore a viable alternative. To avoid enumeration of *all* possible values for  $\varsigma$ , we employ the following scheme. We divide the problem into subproblems by imposing bounds  $0 \leq l \leq u \leq C_{\text{sig}}/b_{\text{sig}}$  on the number of symbol times used for signaling:

$$\max \quad \text{VAR OPT} \quad \text{s.t.} \quad l \leq \varsigma \leq u \tag{VAR' \langle l, u \rangle}$$

We can estimate the value of the optimum and obtain feasible solutions by solving the easier problem (PLAIN) with some additional constraints, that is,

$$\max \quad \text{PLAIN} \quad \text{s.t.} \quad l - 1 < \sum_{j,n} c_{j,n}^{(t)} \cdot x_{j,n}^{(t)} / N \cdot b_{\text{sig}} \leq u \tag{PLAIN \langle l, u \rangle}$$

and applying the relation  $(S - u)\text{PLAIN} \langle l, u \rangle \leq \text{VAR}' \langle l, u \rangle \leq (S - l)\text{PLAIN} \langle l, u \rangle$ . In particular, if  $l = u$ , we obtain the optimal solution. Our approach works as follows:

1. Obtain a starting value  $\varsigma_0$  by computing the number of symbol times used for signaling when using the optimal solution of (PLAIN).

2. Check for better solutions with  $\varsigma_+ = \varsigma_0 + 1, 2, \dots$  symbols spent on signaling by solving  $(\text{VAR}'\langle l, u \rangle)$  with  $l = u = \varsigma_+$ , stopping as soon as  $(S - \varsigma_+) \cdot \text{PLAIN}\langle \varsigma_+, C_{\text{sig}}/b_{\text{sig}} \rangle$  is less than the currently best solution (all higher values of  $\varsigma$  can then be excluded.)
3. Check for better solutions with  $\varsigma_- = \varsigma_0 - 1, 2, \dots$  bits spent on signaling by solving  $(\text{VAR}'\langle l, u \rangle)$  with  $l = u = \varsigma_-$ , stopping as soon as  $S \cdot \text{PLAIN}\langle 0, \varsigma_- \rangle$  is less than the currently best solution value. (All lower values of  $\varsigma$  can then be excluded.)

Empirically, the optimal solution is always very close to  $\varsigma_0$  and enumeration of most possible values for  $\varsigma$  can be skipped by the above method. The solution approach depends on the complexity of problem  $(\text{PLAIN}\langle l, u \rangle)$ . We suspect this problem to be *NP*-hard, but we can not prove it at the moment. In the evaluation, this problem has been optimally solved by using CPLEX.

## 4 Performance Evaluation

The proposed signaling scheme is investigated for an increasing number of sub-carriers. Terminals move across the cell, we compare two different speed values: 1 m/s and 10 m/s. An increased speed reduces the time correlation of sub-carriers. We consider the gross throughput of solving  $(\text{PLAIN})$ , the net throughput for the fixed size signaling phase model and for the variable size signaling phase model. For reference, the throughput of a static (TDMA) scheme is also given. Regarding the detailed methodology we refer the interested reader to [GGKW05].

For the simulations the system model is parameterized as follows. The center frequency is  $f_c = 5.2$  GHz. The system bandwidth is set to  $B = 16.25$  MHz while the frame length is  $T_f = 2$  ms, equally split into up-link and down-link phase. The maximum transmit power is set to  $P_{\text{max}} = 10$  mW.  $J = 8$  terminals are located in the cell. The adaptive modulation scheme is based on  $M = 4$  modulation types (BPSK, QPSK, 16-QAM and 64-QAM) while providing a target symbol error probability of  $10^{-2}$ . The propagation environment parameters (path loss, shadowing and fading) are parameterized according to an large open space environment. For details on the parameterization refer to [GGKW05].

We basically study the following trade-off: For any OFDM system applying a guard interval, increasing the number of sub-carriers under fixed total transmit power and total system bandwidth leads to a throughput increase. The higher the number of sub-carriers, the longer is the symbol duration per sub-carrier. Less guard intervals thus occur per time unit, and the utilization of each sub-carrier is increased, leading to a higher *gross* system throughput. On the other hand, signaling overhead increases, so the *net* throughput is affected differently.

In Figure 1, we observe the first phenomenon directly in the increase of the gross throughput with the number of sub-carriers. Note that the gross throughput advantage of the dynamic scheme is about 50% compared to the static approach. Considering the *net* throughput, however, exposes a different behavior of the system. For a growing number of sub-carriers the signaling overhead increases inevitably. For both signaling schemes, the net throughput thus increases up to a maximum point, and decreases thereafter. At its maximum throughput ( $N = 128$ ), the *fixed size* signaling scheme's overhead amounts to about 10% of the gross throughput. For very high sub-carriers numbers ( $N = 512$ ) the increasing signaling overhead annihilates the performance advantages of the dynamic OFDM scheme.

The variable signaling phase scheme is the only one that is state aware and differs for the two speed parameter values. We see that for low terminal speed of 1 m/s and the resulting high time correlation of sub-carrier attenuation, the signaling overhead is reduced by about 25%.

In case of the variable signaling field scheme exploiting the correlation in time clearly pays off for a

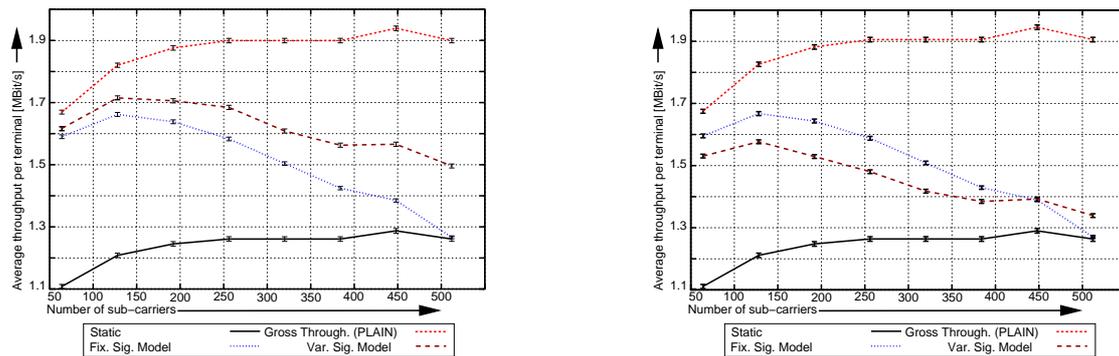


Figure 1: Average throughput per terminal for an increasing number of sub-carriers (fixed bandwidth, fixed transmit power): left  $v_{\max} = 1$  m/s, right  $v_{\max} = 10$  m/s (both with confidence level 0.99).

low mobility. However, at a low correlation in time the performance is well below the one of the fixed signaling field scheme.

## 5 Conclusions

Exploiting the correlation in time in order to decrease the signaling overhead can be quite beneficial for the performance of an dynamic OFDMA system. In case of a high correlation in time the presented scheme has been shown to outperform traditional signaling schemes by about 10%. However, if the correlation in time is low, this performance advantage turns into a performance decrease of about 10%. Thus, in cases with a high time correlation such a scheme pays off well and should be considered for application.

In these cases the performance increase is coupled to a higher computational requirement at the access point. Incorporating the cost of signaling into the sub-carrier assignment problem increases the problem's complexity. While the quadratic nature of the resulting problem can be handled by the presented iterative algorithm, a constrained assignment problem remains to be solved. We consider efficient algorithms either solving this remaining problem optimally or approximately as future work.

## References

- [GGKW05] J. Gross, H. Geerdes, H. Karl, and A. Wolisz. Performance Analysis of dynamic OFDM-FDMA systems with Inband Signaling. Technical Report TKN-05-001, Technische Universität Berlin, 2005.
- [GPKW04] J. Gross, I. Paoluzzi, H. Karl, and A. Wolisz. Throughput Study for a Dynamic OFDM-FDMA System with Inband Signaling. In *Proc. Vehicular Technology Conference (VTC Spring)*, May 2004.
- [KV00] B. Korte and J. Vygen. *Combinatorial Optimization*. Springer, 2000.
- [vNP00] R. van Nee and R. Prasad. *OFDM Wireless Multimedia Communications*. Artech House, 2000.
- [WCLM99] C.Y. Wong, R.S. Cheng, K.B. Letaief, and R. Murch. Multiuser OFDM with Adaptive Subcarrier, Bit and Power Allocation. *IEEE Journal on Selected Areas of Communications*, 17(10):1747–1758, Oct 1999.